

## ECO663

Week 5

## 2. Biases in the evaluation of conjunctive and disjunctive events

- When several events all need to occur to result in a certain outcome we overestimate the likelihood that all of them will happen.
- If only one of many events needs to occur, we underestimate that probability.

- A conjunctive event is comprised of a series of stages where the previous stage must be successful for the next stage to begin.

## Example: Conjunctive Event

- Home remodeling project
- Suppose that each agents (such as carpenters, electricians, plumbers...) will arrive on time 90% of the time.
- What is the probability of completing the project on time?

## Example: Conjunctive Event

Why do home remodeling projects always take longer than planned?

90% chance that the masons, rough carpenters, electricians, plumbers, sheet rockers, finish carpenters, painters, flooring installers, and cabinet installers will each arrive on time.

Unfortunately, this means that the chance that all will be on time is:

- $0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 38\%$ .
- Remember, one late start can ruin the entire chain of arrivals. By the way, if the chance of each showing up on time is 80%, the chance of the entire job running on time is 13%. Lower it to 70% and the chance of a smooth job is 4%—shocking!

- Many people do not think in terms of total event (or system) probability.
- Instead, they anchor on initial stage probabilities and fail to adjust their probability assessment.
- This results in overestimating the likelihood of success for a conjunctive event.

- A disjunctive event occurs in risk assessment.
- When examining complex systems, we may find that the **likelihood of failure of individual critical components or stages is very small**. However, as complexity grows and the **number** of critical components increases, we find mathematically that the probability of event (or system) failure increases.
- However, we again find that people anchor incorrectly. In this case, they **anchor on the initial low probabilities of initial stage failure**.
- Consequently, people frequently underestimate the probability of event failure.

### Example: Disjunctive Event

- There may be a 0.01% chance that an airplane's each of four engines fail, 0.5% of mechanical error, 0.3% of pilot's error, 0.5% of unexpected severe weather condition ....
- Probability of an airplane crash = ?

Example: 3 types of bets: Which one do you prefer, (1) vs. (2) / (1) vs. (3) ?

- (1) **Simple events**, drawing a red marble from a bag containing 50% red marbles and 50% white marbles
- (2) **Conjunctive events**, drawing a red marble seven times in succession, with replacement, from a bag containing 90% red marbles and 10% white marbles
- (3) **Disjunctive event**, drawing a red marble at least once in seven successive tries, with replacement, from a bag containing 10% red marbles and 90% white marbles.

- Preference between (1) and (2) ?
- Preference between (1) and (3)?

- A significant majority of subjects preferred to bet on the conjunctive event rather than the simple event.
- Subjects also preferred to bet on the simple event rather than on the disjunctive event.

- Probabilities?
  - (1) drawing a red from 50% red and 50% white  
= 0.50
  - (2) Drawing a red seven times in succession with replacement, 90% red and 10% white  
=  $(0.9)^7 = 0.478$
  - (3) Drawing at least one red in 7 successive tries, with replacement, 10% red, 90% white  
=  $1 - (0.9)^7 = 0.522$

⇒ People tend to

- overestimate the probability of conjunctive events
- underestimate the probability of disjunctive events.

### 3. Anchoring in the assessment of subjective probability distributions

- Subjects state overly **narrow confidence intervals** which reflect more certainty than is justified by their knowledge about the assessed quantities.

### Example: Dow Jones Index

- Subject is asked to select a number  $X_{90}$  such that his subjective probability that this number will be higher than the value of the Dow-Jones is 0.90.
- By asking  $X_1, X_{10}, X_{25}, X_{50}, X_{75}, X_{99}$ ... subjective cumulative distribution function of the expected index values could be drawn.

⇒ It turned out, the actual probabilities of  $P(X < X_1) + P(X > X_{99}) \approx 0.3$  (while it is "predicted" to be 0.02 (1% each for each tail)).

≤ To select  $X_{99}$ , it is natural to begin by thinking about one's best estimate of the Dow-Jones, and to adjust this value upwards.

≤ To select  $X_1$ , it is natural to begin by thinking about one's best estimate of the Dow-Jones, and to adjust this value downwards.

When the adjustment is insufficient,  $X_{99}$  or  $X_1$  are not sufficiently extreme values ⇒ Narrow confidence interval .

## Representativeness

- Representativeness Heuristics
- Tversky A. and Kahneman D. (1974) Judgment under Uncertainty: Heuristics and Biases, Science, New Series, Vol. 185, No. 4157, pp:1124-1131.
- Kahneman D. and Tversky A. (1972) Subjective Probability: A Judgment of Representativeness, Cognitive Psychology, 3, pp:430-454.
- "Thinking Fast and Slow" Chapters 10, 14, 15, 16,17, 18, 20.

## Representativeness Heuristic

An event A is judged more **probable** than an event B whenever A appears more **representative** than B.

A heuristic that substitutes **probability** with **similarity**.

=> The ordering of events by their **subjective probabilities** coincides with their ordering by **representativeness**.

## Consider a question such as...

- What is the probability that object A belongs to class B?

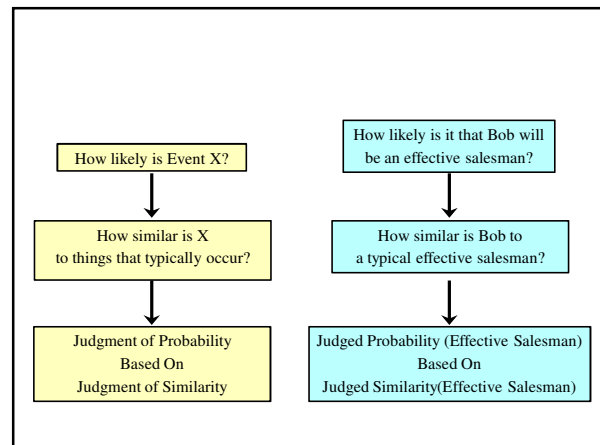
(A: a job candidate, B: successful salesperson)

(A: a person, B: occupation)

Probability

vs.

Similarity



## Representative Heuristics

1. Insensitivity to prior probability of outcomes
2. Insensitivity to sample size
3. Misconceptions of chance
4. The Illusion of validity
5. Misconceptions of regression
6. Conjunction fallacy
7. Dilution effect

### 1. Insensitivity to prior probability of outcomes

- Prior probability = base rate frequency

### What's his occupation?

- Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail.

A: Farmer  
 B: Salesman  
 C: Airplane pilot  
 D: Librarian  
 E: Physician

- The fact that there are more farmers than librarians in the population should enter into the estimate of the probability that Steve is a librarian rather than a farmer.
- If people evaluate probability by representativeness, prior probabilities are neglected.

### Occupation Base Rate in Turkey

- Farmer
- Mining
- Manufacturing
- Service
- Construction
- Trading
- Transportation
- Financial

Tablo 2.2. Sanayi kollarının istihdamdaki payları, 2000 – 2008.

Yıl	Tarım	Maden.	İmalat	Hizm.	İnşaat	Ticaret	Taşınacılık	Finans	Kişisel Hizm.
2000	%36,0	%0,4	%16,9	%0,4	%6,3	%17,7	%4,9	%3,3	%14,1
2008	%24,7	%0,5	%18,6	%0,4	%5,9	%21,6	%5,1	%5,5	%17,3

Kaynak: Toplu TÜİK HİA verileri.

*ILO Yayın Katalogları Verileri*

Ercan, Hakan

Türkiye'de Mesleki Görünüm / Hakan Ercan, Uluslararası Çalışma Örgütü. - Ankara: ILO, 2011

1 v.

ISBN: 9789220256152

ISBN: 9789220256169 (web pdf)

### Exp. Tom W's specialty

- Tom W is a graduate student at the main university in your state. Please rank the following nine fields of graduate specialization in order of the likelihood that Tom W is now a student in each of these fields. Use 1 for the most likely, 9 for the least likely.

Rank Tom W's specialty, 1 the most likely, 9 the least likely.

- Business administration
- Computer science
- Engineering
- Humanities and education
- Law
- Medicine
- Library science
- Physical and life sciences
- Social science and social work

- Tom W is of high intelligence, although lacking in true creativity. He has a need for order and clarity, and for neat and tidy systems in which every detail finds its appropriate place. His writing is rather dull and mechanical, occasionally enlivened by somewhat corny puns and flashes of imagination of the sci-fi type. He has a strong drive for competence. He seems to have little feel and little sympathy for other people, and does not enjoy interacting with others. Self-centered, he nonetheless has a deep moral sense.

Rank Tom W's specially, 1 the most likely, 9 the least likely.

- Business administration
- Computer science
- Engineering
- Humanities and education
- Law
- Medicine
- Library science
- Physical and life sciences
- Social science and social work

- High base rate: humanities and education, social science and social work.
- Low base rate: computer science, engineering
- Base rate = 0 : library science

- Similarity => Stereotype ↔ Probability



Representativeness

<= Question about **probability** was difficult, but the question about **similarity** was easier, and it was answered instead.

### Green cab vs Blue cab problem

- A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:
- 85% of the cabs in the city are Green and 15% are Blue.
- a witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time. What is the probability that the cab involved in the accident was Blue rather than Green?

- Base rate = initial belief  
[ 85% Green cab, 15% Blue cab]
- Updating belief
  1. Heuristic Judgment [Give more weight for new information provided by a witness]
  2. Bayesian Updating

- B: Blue cab caused the accident
- G: Green cab caused the accident
- W: the witness stated "it was Blue cab".

$$P(G|W) = [P(W|G)*P(G)]/P(W)$$

$$P(B|W) = [P(W|B)*P(B)]/P(W)$$

$$P(B|W)/P(G|W) = P(W|B)*P(B) / P(W|G)*P(G)$$

$$= [0.8*0.15] / [0.2*0.85]$$

$$= 12/17$$

Since  $P(B|W)+P(G|W)=1$ ,  $P(B|W)/[1-P(B|W)] = 12/17$ .

=>  $P(B|W) = 0.41$  or 41%.

=> Indicating that despite of the witness testimony, the hit-and-run cab is more likely to be Green than Blue

- Role of Representativeness Heuristic
- Sometimes people overweight new information (more representative or available) and conclude that "Blue" should be the cab.  
[Updating their beliefs with new information heuristically, but not with Bayesian updating]
- On the other hand, people may also overweight prior (previous belief = 85% Green, 15% Blue), and simply ignore new information = conclude that "Green" should be the cab.

### Research Questions

- Conservatism (overweighting the prior)  
Vs.
- Base-rate neglect (overweighting new information)

e.g. Climate Change, Nuclear Power Plants, Food safety, Medical test result....

Q. Do people update their existing beliefs?

Q. If they do, do they do it correctly or heuristically?

Q. If they don't, is there any way to intervene their beliefs?

### Virtues of representative heuristics

- The intuitive impressions often produce more accurate than chance guesses would be.
- A professional athlete who is very tall and thin is much more likely to play basketball than football.
- People with a PhD are more likely to subscribe to The New York Times than people who ended their education after high school.
- Young men are more likely than elderly women to drive aggressively.

### Sin of representativeness heuristics

You see a person reading The New York Times on the New York subway. Which is a better bet?

1. She has a PhD.
2. She does not have a college degree.

- People with a PhD are more likely to subscribe to The New York Times than people who ended their education after high school.

- People with a PhD < without PhD
- People without PhD ride New York subways more often.

<= If you ignore the second fact, make a wrong judgment.

## 2. Insensitivity to sample size

- To evaluate the probability of obtaining a particular result in a sample drawn from a specified population, people often ignore the effect of sample size.
- E.g. The probability of obtaining an average height greater than 180 cm is assigned the same value for samples of 10, 100 or 1000 men.

- The **size of a sample** withdrawn from a population should greatly affect the likelihood of obtaining certain results in it
- People, however, ignore sample size and only use the superficial similarity measures
- For example, people ignore the fact that **larger** samples are **less** likely to deviate from the mean than **smaller** samples

- Intuitive sampling distributions completely ignore effect of sample size on variance.
- Law of Large Numbers: The larger the sample, the higher the probability that an estimate of the mean will be close to the true mean.
- Estimates based on small samples are inferior to estimates based on large samples, but this way of asking for the estimate shows no awareness of this.

## e.g. % of boy babies

- A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Some times it may be higher than 50 percent, sometimes lower.
- For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?
  - A: The larger hospital
  - B: The smaller hospital
  - C: About the same

A: The larger hospital (21)  
 B: The smaller hospital (21)  
 C: About the same (53)

( ): answers by undergraduate students in the experiment.

=> The smaller hospital because a large sample is less likely to stray from 50 %.

- It is easier to get 6 heads with 10 flips of a coin than 6,000 heads with 10,000 flips of a coin



### 3. Misconceptions of chance

- **Irregularity** and **local representativeness** seem to capture the intuitive notion of randomness.

(Truth) Law of large numbers: very large samples are highly representative of the populations from which they are drawn.

(Belief) Law of small numbers: The expectancy of local representativeness

(Belief) => "The law of large numbers applies to small numbers as well"

- Which pattern is more likely as the result of 6 coin flips? (H- head, T-tail)

1: H-T-H-T-T-H

2: H-H-H-T-T-T

3: H-H-H-H-T-H

⇒ They are all equally likely.

⇒ Randomness observed in large sample, may not appear "random" in the short sequences (although we expect it should look "random" in short sequences as well => Error).

#### Beliefs about random event:

- Random events are (invariably) *patternless*.
- Events that display patterns are not random  
=<= they have underlying causes.

#### EXAMPLES

- Random coin flips should look like...  
HTHTTHTHH ....

- People expect that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short (local representativeness ⇔ global).

- Chance is commonly viewed as a self-correcting process in which a deviation in one direction (e.g. too many Head) induces a deviation in the opposite direction to restore the equilibrium (Belief, not Truth). => Gambler's Fallacy

#### e.g. Gambler's Fallacy

What is the probability of having H after 4 consecutive Hs?

1	2	3	4	5
H	H	H	H	?

$$P(\text{HHHH}) = (1/2)^4 = 1/16 = 0.0625$$

$$P(\text{H} | \text{HHHH}) = ?$$

$$P(\text{H} | \text{HHHH}) = 1/2$$

$$\text{While } P(\text{HHHHH}) = (1/2)^5 = 1/32 = 0.03125$$

- But people believe that if something happens more frequently than normal during some period, then it will happen less frequently in the future.

=> Expect more chance of T than H, bet on T although the chance of 5<sup>th</sup> H is the same as the chance of T in the 5<sup>th</sup> run.

- **Independence** – what happens in the past has no influence over what happens next.
- **Stationary** – the probability of the event doesn't change over time.

Examples:

- Chance of "heads" when flipping a coin is independent and stationary.
- Chance of rolling a "2" with a die is independent and stationary.

#### 4. The Illusion of Validity

- People often predict by selecting the outcome (e.g. successful businessperson) that is most representative of the input (e.g. the description of a person).
- The **confidence** they have in their prediction depends primarily on the degree of representativeness **with little or no regard for the factors that limit predictive accuracy.**

- The unwarranted confidence which is produced by a good fit between the predicted outcome and the input information => Illusion of Validity

#### examples

- Job interview (successful future stock dealer? executive candidate? Innovative?...)
- Student selection by interview (graduate with high GPA? Does good research? Candidate of future faculty?...)
- Predicting students' final GPA based on 1<sup>st</sup> year record (all B vs. many As and Cs).
- Predicting future stock prices (individual investors)
- Selection of potentially successful leaders

Input:

- Unreliable information,
- Insufficient information
- Information with redundant or correlated variables
- Information on irrelevant attributes
- Available information (influence of media)

Output: Poor prediction of the future

## 6. Conjunction Fallacy

- occurs when it is assumed that specific conditions are more probable than a single general one.

### Linda Problem

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Rank the following options, 1 being the most probable and 8 being the least probable.

- A teacher in elementary school
- Works in a bookstore and takes yoga classes
- Active in the feminist movement
- A psychiatric social worker
- A member of the League of Women Voters
- A bank teller
- An insurance sales person
- A bank teller and is active in the feminist movement

What's your ranking of 6 (bank teller) and 8 (feminist bank teller)? Is 8 higher in the rank than 6? => Conjunction Fallacy

**T** = Linda is a bank teller.  
 $P(T)$  = Probability of Statement T

**F** = Linda is active in the feminist movement.  
 $P(F)$  = Probability of Statement F

**TF = T&F** = Linda is a bank teller and is active in the feminist movement.  
 $P(T \cap F)$  = Probability of Statement T&F

If  $P(T \cap F)$  is ranked higher than  $P(T)$ , something is wrong! WHY?

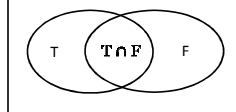
### Conjunction

**Principle:** The probability of a conjunction of events is always equal or less than the probability of either event in the conjunction. I.e., mathematics tells us that the following are both true:

$$P(T \cap F) \leq P(T),$$

$$P(T \cap F) \leq P(F)$$

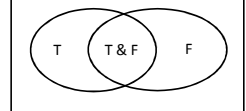
Sample Space (set of all possibilities; not set of all features)



### "Linda" Problem: (Description).

- T: Linda is a bank teller.
- F: Linda is a feminist.
- T & F: Linda is a bank teller who is active in the feminist movement.

Sample Space (set of all possibilities; not set of all features)



- Probability Theory:  
 $P(F) > P(F \cap T)$ ,  $P(T) > P(F \cap T)$
- Paradoxical finding:  $JP(F) > JP(F \cap T) > JP(T)$
- "bank teller & feminist" is a subset of "bank teller." Therefore it MUST have a lower probability than "bank teller."

### Why Do People Make Conjunction Errors?

Kahneman & Tversky's Answer to this Question:

- People substitute similarity judgment for probability judgment.
- Human intuitions of similarity differ from the mathematical structure of probability.
- These differences produce errors in probabilistic reasoning.

### Competing Arguments for Probabilistic Reasoning and Representativeness

- Probability Theory: Linda is more likely to be a bank teller than she is to be a feminist bank teller, because every feminist bank teller is a bank teller, but some women bank tellers are not feminists, and Linda could be one of them.
- Representativeness: Linda is more likely to be a feminist bank teller than she is likely to be a bank teller, because she resembles an active feminist more than she resembles a bank teller.

65% prefer the representativeness argument over the probability theory argument.

### 7. Dilution effect

- Combining non-diagnostic information with diagnostic information makes an outcome seem less probability.
- Explanation: Non-diagnostic information makes the current case less similar to typical cases.

	Condition 1	Condition 2
Diagnostic Info	David is sexually aroused by violent sadomasochistic fantasies. David has a serious drinking problem.	David wants to adopt a second child. He does volunteer work at a neighborhood school to promote good race relations.
Non-Diagnostic Info	David has an IQ of 110. He injured his back in a skiing accident. David is strong-minded and is rarely willing to back off on an issue of principle. David likes to tell jokes.	
Probability Judgment	Rate how likely it is that David is a child abuser (scale 1 to 10).	

Tetlock, P. E., & Boettger, R. (1989). Accountability: A social magnifier of the dilution effect. *Journal of Personality and Social Psychology*, 57(3), 388-398.

**Dilution Effect:**  
Non-diagnostic information reduces the impact of diagnostic information.

