ECO611, HW 4 Questions Eigenvalues, Eigenvectors, Differential Calculus

1. Find eigen values and eigen vectors for the each matrix. Show that $P^{-1}AP = \Lambda$. Refer to lecture note for the definition of P, A and Λ.

(a) $A = \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 5 & 6 \\ -1 & -2 \end{bmatrix}$

2. A person's productivity at a particular job may change with experience. Consider the following model of effective labor input, L, $L = 10(1 - e^{-0.1t}) + 2$, where L is a measure of the "effective labor input" of a particular worker and t is the worker's tenure (that is, years spent in a particular job). Use differentiation to determine the marginal change in the effective labor input with respect to time on the job, t. Provide a brief economic interpretation of the differences in these values of dL/dt.

3. The law of diminishing marginal returns states that the incremental output obtained from additional units of a variable input, if all other inputs are held constant, decreases as more of the variable input is added. Geometrically, this means that the slope of the total product curve is decreasing and the slope of the marginal product curve is negative. This can be determined by taking the derivative of the marginal product function, which is the same as taking the second derivative of the total product function. Consider the following short-run production function in which capital is held constant and labor is the variable input: $Q = f(K_0, L) = 50K^{1/3}L^{2/3}$. Assume that capital is held constant at K = 27. Calculate the total product and the marginal product of labor. Can you determine if this production function exhibits diminishing marginal returns to labor?

4. Find the 3rd degree Taylor approximation for $f(x) = e^x$ around the point x = 0.

5. If $u = \ln(v^4 + x^4 + y^4 + z^4 - 4vxyz)\,, show\ that\ v\frac{\partial u}{\partial v} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 4$.

6. Determine if the function is homogeneous, and if so, of what degree. $f(x, y, w) = \sqrt{x^2 + y^2}$.

7. Consider the production function $y = f(K, L) = K^{1/2}L^{1/4}$

(a) Determine if this production function is homogeneous. If so, of what degree?
(b) Take the partial derivatives of the production function, and show that they are homogeneous of degree k -1.
(c) Using Euler's Theorem, show that $x_1 f_1(sx_1, sx_2) + x_2 f_2(sx_1, sx_2) = ks^{k-1}f(x_1, x_2)$.

8. Show that each function is homothetic by transforming it back to its original homogenous form.

(a) $y = 0.3\ln(L) + 0.7\ln(K)$,
(b) $y = 2\ln(x) + \ln(y) - \ln(w)$

9. Consider the production function $Q = x_1^{0.25}x_2^{0.75}$. Find the slope of the isoquant and determine the degree of homogeneity.