

ECO611 Homework 3 Questions for Chapters 4 and 5 "Matrix Algebra"

Due: Nov. 8, 2012 at 13:00

1. Consider an example of a country which produces two goods, wheat (W) and steel (S), using two factors of production, labor (L) and land (K). The technical coefficients of production a_{ij} are

$$a_{LS} = 12, a_{KS} = 4, a_{LW} = 2, a_{KW} = 6$$

where i represents the amount of input required to produce good j . The zero-profit condition for these two industries, divided by total output is

$$\frac{L}{S}w + \frac{K}{S}r = p_s \quad \text{and} \quad \frac{L}{W}w + \frac{K}{W}r = p_w$$

where w is the wage paid to labor and r is the rent paid to capital. This system can be expressed in matrix format as $Aw = p$, where A is the matrix of technical coefficients, w is the return paid to the factors of production, and p is the vector of prices.

- (a) Set up this system of equations in matrix format.
- (b) What is the determinant of the matrix of technical coefficients? What does the sign of the determinant indicate about the relative intensity of labor usage in the production processes of both goods?
- (c) Find the solution of this system of equations by using matrix inversion.
- (d) Now assume that the price of steel $P_s = 128$ and the price of wheat $P_w = 48$. What are the resulting values of w and r ?
- (e) What is the impact on w and r if P_w rises to 64 ($\Delta P_w = 16$)? Can you provide an intuition for your result?

2. A standard IS/LM model can be expressed as the 4 x 4 matrix system H ,

$$\begin{bmatrix} 0 & 1 & 0 & d \\ -1 & -1 & 1 & 0 \\ 1 & 0 & -b & 0 \\ 0 & 0 & k & -j \end{bmatrix} \cdot \begin{bmatrix} C \\ I \\ Y \\ R \end{bmatrix} = \begin{bmatrix} e \\ G \\ a \\ M/P \end{bmatrix}$$

where lowercase letters represent parameters and uppercase letters represent variables. The variables in the 4 X 1 vector v , consumption (C), investment (I), income (Y), and interest rates (R), are endogenous. The variables in the 4 X 1 vector p are exogenous and represent government spending (G) and the real money supply (M/P). The first row of H represents the investment equation, the second row represents the national income accounting identity, the third row represents the consumption equation, and the fourth row represents money demand. Find the determinant of this system. Solve for Y^* .

3. Consider the export/import model

$$X = 1000 - 20E + 0.2Y_F$$

$$M = 450 - 10E + 0.15Y_D$$

$$X = M$$

where X represents exports, M represents imports, E represents the exchange rate, Y_F represents foreign income, and Y_D represents domestic income. The exogenous variables include Y_F and Y_D , and the endogenous variables are X , M and E .

- (a) Set up this as a system of equations, and solve this system by finding the inverse of the matrix of parameters.
- (b) Solve this by using Cramer's Rule.
- (c) Determine the impact on exports of a \$100 increase in foreign income, *ceteris paribus*.