ECO 241 HW Questions for Chapter 5.

1. Are the rows linearly independent in each of the following?

(a) $\begin{bmatrix} 1 & 8 \\ 9 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 4 \\ 2 & -8 \end{bmatrix}$

2. Are the columns linearly independent in each of the above (a) – (d)?

3. Evaluate the following determinants:

(a) $\begin{vmatrix} 8 & 1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 1 & 4 \\ 8 & 11 & -2 \\ 0 & 4 & 7 \end{vmatrix}$

4. Evaluate the following determinant:

(a) $\begin{vmatrix} 1 & 2 & 0 & 9 \\ 2 & 3 & 4 & 6 \\ 1 & 6 & 0 & -1 \\ 0 & -5 & 0 & 8 \end{vmatrix}$

(b) Find the value of the cofactor of the element 9.

5. Which properties of determinants enable us to write the following?

(a) $\begin{vmatrix} 9 & 18 \\ 27 & 56 \end{vmatrix} = \begin{vmatrix} 9 & 18 \\ 0 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 9 & 27 \\ 4 & 2 \end{vmatrix} = 18 \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$

6. Test whether the following matrices are nonsingular:

(a) $\begin{bmatrix} 4 & 0 & 1 \\ 19 & 1 & 3 \\ 5 & 4 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6 \end{bmatrix}$

7. Rewrite the simple national income model

$$Y = C + I_0 + G_0$$
$$C = a + bY \qquad (a>0,\ 0<b<1)$$

in the $Ax = d$ format where $x = \begin{bmatrix} Y \\ C \end{bmatrix}$

and test whether the coefficient matrix A is nonsingular.

8. Find the inverse of the following matrices:

(a) $A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 0 \\ 9 & 2 \end{bmatrix}$

(c) $C = \begin{bmatrix} 4 & -2 & 1 \\ 7 & 3 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ (d) $D = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$

9. Use Cramer's Rule to solve the following equation systems.

(a)
$$8x_1 - x_2 = 15$$
$$x_2 + 5x_3 = 1$$
$$2x_1 + 3x_3 = 4$$

(b)
$$-x_1 + 3x_2 + 2x_3 = 24$$
$$x_1 + x_3 = 6$$
$$5x_2 - x_3 = 8.$$

10. Solve the national income model

$$Y = C + I_0 + G_0$$
$$C = a + b(Y - T) \qquad (a>0,\ 0<b<1)$$
$$T = d + tY \qquad (d>0,\ 0<t<1)$$

(a) by matrix inversion (b) by Cramer's rule.

where $x = \begin{bmatrix} Y \\ C \\ T \end{bmatrix}$

11. Solve the national income model

$$Y = C + I_0 + G$$
$$C = a + b(Y - T_0) \qquad (a>0,\ 0<b<1)$$
$$G = gY \qquad (0<g<1)$$

(a) by matrix inversion (b) by Cramer's rule

where $x = \begin{bmatrix} Y \\ C \\ G \end{bmatrix}$

12. Leontief Input-Output Model

Given $A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}$ and $d = \begin{bmatrix} 30 \\ 15 \\ 10 \end{bmatrix}$ (final demand)

(a) Solve for X_1^*, X_2^* & X_3^*.

(b) Calculate the total amount of primary input required to produce the solution outputs.

13. In a two-industry economy, it is known that industry I uses 10 cents of its own product and 60 cents of commodity II to produce a dollar's worth of commodity I; Industry II uses none of its own product but uses 50 cents of Commodity I in producing a dollar's worth of Commodity II; and the open sector demands \$ 1000 billion of Commodity I and \$ 2000 billion of Commodity II.

(a) Write out the input matrix (A), the technology matrix (I-A) and the specific input-output matrix equation for this economy.

(b) Find the solution output levels by Cramer's rule

14. Given the input matrix and the final demand vector

$A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix}$ $d = \begin{bmatrix} 1800 \\ 200 \\ 900 \end{bmatrix}$

(a) Explain the economic meaning of the elements 0.33, 0, and 200.

(b) Explain the economic meaning of the third-column sum.

(c) Write out the specific input-output matrix equation for this model.

(d) Find the solution output levels of the three industries by Cramer's rule.

(Round-off answers to two decimal places)