

# HW Questions for Chapter 10.

1. Find extreme values of the following functions & determine whether they are maximum/minimum.

a.  $Z = X^2 + XY + 2Y^2 + 3$

b.  $Z = aX^2 + bY^2 + c$ : consider each of the three subcases:

(a)  $a > 0, b > 0$ , (b)  $a < 0, b < 0$ , (c)  $a$  and  $b$  opposite in sign.

c.  $Z = e^{2X} - 2X + 2Y^2 + 3$ .

2. Consider the function  $Z = (X-2)^4 + (Y-3)^4$

a. Establish by intuitive reasoning that  $Z$  attains a minimum  $Z^* = 0$  at  $\bar{X} = 2$  and  $\bar{Y} = 3$ .

b. Is the F.O.C. satisfied?

c. Is the S.O.C. satisfied?

d. Find the value of  $d^2Z$ . Does it satisfy the second-order conditions for a minimum?

3. Determine whether  $q = 2u^2 + 3v^2 - w^2 + 6uv - 8uw - 2vw$  is either positive or negative definite.

4. Use Definition 1 (without any derivative) to check whether the following functions are concave, convex, strictly concave, strictly convex, or neither.

a.  $Z = X^2$

b.  $Z = X_1^2 + X_2^2$

5. Use Definition 2 (w/ 1st derivative) to check whether the following functions are concave, convex, strictly concave, strictly convex, or neither.

a.  $Z = -X^2$

b.  $Z = (X_1 + X_2)^2$

c.  $Z = -XY$

6. A two-product firm under perfect competition. (2)

Revenue function:  $R = P_1 Q_1 + P_2 Q_2$

Cost function:  $C = 2Q_1^2 + 2Q_2^2$

a. What will be the new optimal levels of  $Q_1$  &  $Q_2$

b. What's the value of  $\pi$ ?

7. A two-product firm faces the demand and cost functions below:

$$Q_1 = 40 - 2P_1 - P_2$$

$$Q_2 = 35 - P_1 - P_2$$

$$C = Q_1^2 + 2Q_2^2 + 10$$

a. Find the output levels that satisfy the F.O.C. for maximum profit.

b. Check the S.O.C.

c. What's the maximum profit?