Hypothesis Tests for the difference between the population means:

Dependent Samples (Paired samples)

$H_0: \mu_x - \mu_y \leq D_0$ $H_1: \mu_x - \mu_y > D_0$	$Upper\ tail\ test: Reject\ H_0\ if\ \bar{d} > D_0 + t_{n-1,\alpha} \frac{sd}{\sqrt{n}}$
$H_0: \mu_x - \mu_y \geq D_0$ $H_1: \mu_x - \mu_y < D_0$	$Lower\ tail\ test: Reject\ H_0\ if\ \bar{d} < D_0 - t_{n-1,\alpha} \frac{sd}{\sqrt{n}}$
$H_0: \mu_x - \mu_y \neq D_0$ $H_1: \mu_x - \mu_y = D_0$	$Two\ tail\ test\ (upper\ tail\ case\ if\ \bar{d} - D_0 > 0):$ $Reject\ H_0\ if\ \bar{d} > D_0 + t_{n-1,\frac{\alpha}{2}} \frac{sd}{\sqrt{n}}$
	$Two\ tail\ test\ (lower\ tail\ case\ \ if\ \bar{d} - D_0 < 0):$ $Reject\ H_0\ if\ \bar{d} < D_0 - t_{n-1,\frac{\alpha}{2}} \frac{sd}{\sqrt{n}}$

Independent Samples

Case1: σ_x^2 and σ_y^2 known.

$H_0: \mu_x - \mu_y \leq D_0$ $H_1: \mu_x - \mu_y > D_0$	$Upper\ tail\ test: Reject\ H_0\ if\ \bar{d} > D_0 + z_\alpha \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$
$H_0: \mu_x - \mu_y \geq D_0$ $H_1: \mu_x - \mu_y < D_0$	$Lower\ tail\ test: Reject\ H_0\ if\ \bar{d} < D_0 - z_\alpha \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$
$H_0: \mu_x - \mu_y \neq D_0$ $H_1: \mu_x - \mu_y = D_0$	$Two\ tail\ test\ (upper\ tail\ case\ if\ \bar{d} - D_0 > 0):$ $Reject\ H_0\ if\ \bar{d} > D_0 + z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$
	$Two\ tail\ test\ (lower\ tail\ case\ \ if\ \bar{d} - D_0 < 0):$ $Reject\ H_0\ if\ \bar{d} < D_0 - z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$

Case2: σ_x^2 and σ_y^2 unknown. $Assumed\ \sigma_x^2 = \sigma_y^2$.

$H_0: \mu_x - \mu_y \leq D_0$ $H_1: \mu_x - \mu_y > D_0$	$Upper\ tail\ test: Reject\ H_0\ if\ \bar{d} > D_0 + t_{nx+ny-2,\alpha} \sqrt{\frac{S_p^2}{n_x} + \frac{S_p^2}{n_y}}$
$H_0: \mu_x - \mu_y \geq D_0$ $H_1: \mu_x - \mu_y < D_0$	$Lower\ tail\ test: Reject\ H_0\ if\ \bar{d} < D_0 - t_{nx+ny-2,\alpha} \sqrt{\frac{S_p^2}{n_x} + \frac{S_p^2}{n_y}}$
$H_0: \mu_x - \mu_y \neq D_0$ $H_1: \mu_x - \mu_y = D_0$	$Two\ tail\ test\ (upper\ tail\ case\ if\ \bar{d} - D_0 > 0):$ $Reject\ H_0\ if\ \bar{d} > D_0 + t_{nx+ny\ \ ,\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_x} + \frac{S_p^2}{n_y}}$
$where\ S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{(n_x + n_y - 2)}$	$Two\ tail\ test\ (lower\ tail\ case\ \ if\ \bar{d} - D_0 < 0):$ $Reject\ H_0\ if\ \bar{d} < D_0 - t_{nx+ny-2,\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_x} + \frac{S_p^2}{n_y}}$

Case3: σ_x^2 and σ_y^2 unknown.

<table>
<tr>
<td>$H_0: \mu_x - \mu_y \leq D_0$
$H_1: \mu_x - \mu_y > D_0$</td>
<td>$Upper\ tail\ test: Reject\ H_0\ if\ \bar{d} > D_0 + t_{v,\alpha}\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$</td>
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<td>$H_0: \mu_x - \mu_y \geq D_0$
$H_1: \mu_x - \mu_y < D_0$</td>
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</tr>
<tr>
<td rowspan="2">$H_0: \mu_x - \mu_y \neq D_0$
$H_1: \mu_x - \mu_y = D_0$
$where\ v = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x - 1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{(n_y - 1)}}$</td>
<td>$Two\ tail\ test\ (upper\ tail\ case\ if\ \bar{d} - D_0 > 0):$
$Reject\ H_0\ if\ \bar{d} > D_0 + t_{v,\frac{\alpha}{2}}\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$</td>
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