

Q11.

X	0	4	14	10	7	8	6	1	$\bar{X} = 6.5$
Y	2.4	7.2	10.3	9.1	10.2	4.1	7.6	3.5	$\bar{Y} = 6.8$

$(X_i - \bar{X})^2$	42.25	6.25	56.25	12.25	6.25	2.25	0.25	30.25	$\sum (X_i - \bar{X})^2 = 156$
$(Y_i - \bar{Y})^2$	0.64	3.04	0.49	0.81	0.64	1.21	0.36	0.25	$\sum (Y_i - \bar{Y})^2 = 22.10$
$(X_i - \bar{X})(Y_i - \bar{Y})$	-2.6	-1	26.25	8.05	8.5	-4.05	-0.4	12.15	$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 24.1$
$\hat{Y}$	3.30	5.45	10.44	8.69	8.15	7.61	6.53	3.43	

$b_1 = \frac{24.1}{156} = 0.154$ a) $\hat{Y} = 3.30 + 0.154 X$

$b_0 = \bar{Y} - b_1 \bar{X} = 6.8 - 0.154 \times 6.5 = 3.30$

$se^2 = \frac{\sum (Y_i - \hat{Y})^2}{n-2} = \frac{22.10}{6} = 3.68$

$sb_1^2 = \frac{se^2}{\sum (X_i - \bar{X})^2} = \frac{3.68}{156} = 0.024$

b) $b_1 - t_{n-2, \alpha/2} \cdot sb_1 < \beta_1 < b_1 + t_{n-2, \alpha/2} \cdot sb_1$
 $0.154 - \frac{t_{6, 0.05}}{1.943} \cdot \sqrt{0.024} < \beta_1 < 0.154 + \frac{t_{6, 0.05}}{1.943} \cdot \sqrt{0.024}$

$0.239 < \beta_1 < 0.241$

Q12 $\hat{Y}_0 = 14 + 7X_0$
 $se = 7.45$
 $\bar{X} = 8$
 $n = 25$
 $\sum (X_i - \bar{X})^2 = 300$

$X = 11$

o Confidence interval (95%)

$\hat{Y}_{n+1} \pm t_{n-2, \alpha/2} \sqrt{\left(\frac{1}{n} + \frac{(X_{n+1} - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right)} \cdot se$
 $\frac{14 + 7 \times 11}{91} \pm \frac{t_{23, 0.025}}{2.069} \sqrt{\left(\frac{1}{25} + \frac{(11 - 8)^2}{300}\right)} \cdot 7.45$

o predictive interval $91 \pm 4.078 \rightarrow [86.922 \quad 95.078]$

$\hat{Y}_{n+1} \pm t_{n-1, \alpha/2} \sqrt{\left(1 + \frac{1}{n} + \frac{(X_{n+1} - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right)} \cdot se$
 $91 \pm 2.069 \sqrt{\left(1 + \frac{1}{25} + \frac{(11 - 8)^2}{300}\right)} \cdot 7.45$
 $91 \pm 15.94 \rightarrow [75.06 \quad 106.94]$

Q13-1

$$\bar{X} = 1.04$$

$$\bar{Y} = 18$$

Dosage level (X)	1.2	1.3	1.0	1.4	1.5	1.8	1.2	1.3	1.4	1.3
Recovery time (Y)	25	28	40	38	10	9	27	30	16	18

$(X - \bar{X})^2$	0.020	0.002	0.116	0.004	0.026	0.212	0.020	0.002	0.004	0.002
$(X - \bar{X})(Y - \bar{Y})$	-0.126	-0.156	-5.406	0.834	-2.356	-6.946	-0.446	-0.216	-0.446	-0.244
$\hat{Y}$	29.28	25.58	36.67	21.88	18.18	7.09	29.28	25.58	21.88	25.58
$(Y - \hat{Y})^2$	18.29	5.86	11.07	259.82	66.96	3.65	3.19	19.54	34.59	57.44

$$b_1 = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sum (X - \bar{X})^2} = \frac{-14.94}{0.404} = -36.98$$

$$b_0 = 18 - (-36.98) \times 1.04 = 73.65$$

$$a) \quad \hat{Y} = 73.65 - 36.98X$$

$$s_e^2 = \frac{\sum (Y_i - \hat{Y})^2}{n - 2} = \frac{482.42}{8} = 60.30$$

$$s_{b_1}^2 = \frac{s_e^2}{\sum (X_i - \bar{X})^2} = \frac{60.30}{0.404} = 149.26$$

$$s_{b_1} = 12.22$$

$$b) \quad b_1 - t_{n-2, 0.05} \cdot s_{b_1} < \beta_1 < b_1 + t_{n-2, 0.05} \cdot s_{b_1}$$

$$-36.98 - 1.860 \cdot 12.22 < \beta_1 < -36.98 + 1.860 \cdot 12.22$$

$$-59.71 < \beta_1 < -14.25$$

The probability is 0.90 that the interval -59.71 to -14.25 includes the true slope coefficient.

c) No. The prediction is outside the range of the data used for estimation. There is no guarantee that the same relationship holds outside the range of the data and predictions cannot be made for a dosage of 2.0 grams.

* Add $R^2 = 0.7662$ to the setting.

Q13-2.

$$\begin{aligned} n &= 20 \\ \sum y_i &= 22.6 \\ \sum x_i &= 25.4 \\ \sum x_i^2 &= 145.7 \\ \sum x_i y_i &= 150.5 \\ \sum y_i^2 &= 196.2 \end{aligned}$$

$$SST = \sum (y_i - \bar{y})^2 = \sum y_i^2 - n\bar{y}^2 = 196.2 - 20 \times \left(\frac{22.6}{20}\right)^2 = 170.662$$

$$SSE = SST(1 - R^2) \quad \left(\leftarrow R^2 = 1 - \frac{SSE}{SST} \right)$$

$$R^2 = 0.7662$$

$$SSE = 170.662(1 - 0.7662) = 39.9008$$

$$Se^2 = \frac{39.9008}{18} = 2.2167$$

a. $H_0: \beta_1 = 0$

$H_1: \beta_1 > 0$

Reject H_0 if $t_{n-2, \alpha} < t_c = \frac{b_1}{sb_1}$

* Test at 1% level

$$sb^2 = \frac{Se^2}{\sum (x_i - \bar{x})^2} = \frac{Se^2}{\sum x_i^2 - n\bar{x}^2} = \frac{2.2167}{145.7 - 20\left(\frac{25.4}{20}\right)^2} = 0.0195$$

$$sb = 0.1396$$

$$t_c = \frac{b_1}{0.1396} = \frac{1.0737}{0.1396} = 7.69$$

$$t_{18, 0.01} = 2.552$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\sum x_i y_i - \bar{y} \sum x_i - \bar{x} \sum y_i + n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

$$= \frac{150.5 - 2.26 \times 25.4 - 0.57 \times 145.7 + 20 \times 0.57 \times 2.26}{145.7 - 20 \times (2.26)^2} = 1.0737$$

Since

$$t_{18, 0.01} = 2.552 < t_c = 7.69$$

Reject H_0

b. $H_0: \beta_1 = 1$

$H_1: \beta_1 \neq 1$

* Test at 1% level

$$t_{18, 0.005} = 2.878$$

$$t_c = \frac{b_1 - \beta_1}{sb_1} = \frac{1.0737 - 1}{0.1396} = 0.5279$$

Reject H_0 if $t_{18, 0.005} < t_c$

Since $t_{18, 0.005} = 2.878 > t_c = 0.5279$,
we fail to reject H_0 .

Q14.

$$n = 25$$

(10)

part b
is
different!

$$\hat{y} = 15.6 + 1.3X$$

$$SST = 268$$

$$SSE = 204$$

a.

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{204}{268} = 0.2388.$$

23.88% of the variation in the dependent variable can be explained by the variation in the independent var.

b.

$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

Reject H_0 if $t_{n-2, 0.025} < t_c$.

or

$$-t_{n-2, 0.025} > t_c$$

$$t_{23, 0.025} = 2.069$$

$$t_c = \frac{b_1}{Sb_1} = \frac{1.3}{\sqrt{0.234}} = 2.687$$

$$Sb_1 = \sqrt{\frac{Se^2}{\sum (X_i - \bar{X})^2}} = \sqrt{\frac{SSE/(n-2)}{\sum (X_i - \bar{X})^2}} = \sqrt{\frac{204/23}{\sum (X_i - \bar{X})^2}} = \sqrt{\frac{204/23}{37.87}} = \sqrt{0.234}$$

$$\sum (X_i - \bar{X})^2 = \dots$$

$$R^2 = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \frac{b_1^2 \sum (X_i - \bar{X})^2}{\sum (y_i - \bar{y})^2}$$

$$\left(\frac{SSR}{SST} \right)$$

$$\begin{aligned} \hat{y}_i - \bar{y} &= b_0 + b_1 X_i - \bar{y} \\ &= \bar{y} - b_1 \bar{X} + b_1 X_i - \bar{y} \\ &= b_1 (X_i - \bar{X}) \end{aligned}$$

$$\sum (\hat{y}_i - \bar{y})^2 = b_1^2 \sum (X_i - \bar{X})^2$$

Solve for $\sum (X_i - \bar{X})^2$.

$$\sum (X_i - \bar{X})^2 = \frac{R^2 \sum (y_i - \bar{y})^2}{b_1^2}$$

$$= \frac{0.2388 \times 268}{(1.3)^2}$$

$$= 37.87$$

$\Rightarrow$ Since $t_{23, 0.025} = 2.069 < t_c = 2.687$

we reject H_0

$$c) b_1 \pm t_{23, 0.025} Sb_1 \Rightarrow 1.3 \pm 2.069 \times \sqrt{0.234} \Rightarrow [0.299 \quad 2.301]$$