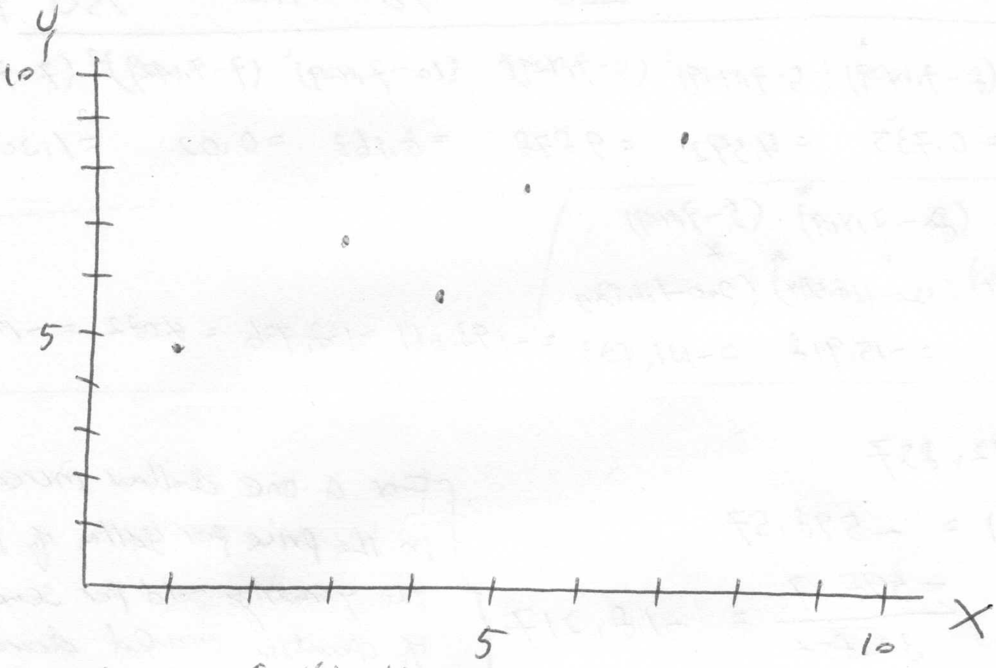
ECO 240 HW Answers for Ch. 11 (Two Variable Regression Analysis) ①

Q1 (1,5) (3,7) (4,6) (5,8) (7,9)



b) $b_1 = \frac{Cov(X,Y)}{S_X^2} = \frac{\Sigma(X_i-\bar{X})(Y_i-\bar{Y})/(n-1)}{\Sigma(X_i-\bar{X})^2/(n-1)} = \frac{13}{20} = 0.65$

X	1	3	4	5	7	$\bar{X} = 4$
Y	5	7	6	8	9	$\bar{Y} = 7$
$(X_i-\bar{X})^2$	$(1-4)^2 = 9$	$(3-4)^2 = 1$	$(4-4)^2 = 0$	$(5-4)^2 = 1$	$(7-4)^2 = 9$	$\Sigma(X_i-\bar{X})^2 = 20$
$(X_i-\bar{X}) * (Y_i-\bar{Y})$	$(1-4)*(5-7) = 6$	$(3-4)*(7-7) = 0$	$(4-4)*(6-7) = 0$	$(5-4)*(8-7) = 1$	$(7-4)*(9-7) = 6$	$\Sigma(X_i-\bar{X})(Y_i-\bar{Y}) = 13$

c) $b_0 = \bar{Y} - b_1\bar{X} = 7 - 0.65 * 4 = 4.4$

d) $\hat{Y} = 4.4 + 0.65X$

Q2. $n = 7$

X	10	8	5	4	10	7	6	$\bar{X} = 7.1429$
Y	100	120	200	200	90	110	150	$\bar{Y} = 138.5714$

$(X_i - \bar{X})^2$: $(10-7.1429)^2$ $(8-7.1429)^2$ $(5-7.1429)^2$ $(4-7.1429)^2$ $(10-7.1429)^2$ $(7-7.1429)^2$ $(7-7.1429)^2$

$= 8.163$ $\quad = 0.735$ $\quad = 4.592$ $\quad = 9.878$ $\quad = 8.163$ $\quad = 0.02$ $\quad = 1.306$

$(X_i - \bar{X}) * (Y_i - \bar{Y})$: $(10-7.1429) * (100-138.5714)$ $\quad (8-7.1429) * (120-138.5714)$ $\quad (5-7.1429) * (200-138.5714)$

$= -110.204$ $\quad = -15.918$ $\quad = -131.633$ $\quad = -193.061$ $\quad = -138.776$ $\quad = 4.082$ $\quad = -18.061$

$\Sigma(X_i - \bar{X})^2 = 32.857$

$\Sigma(X_i - \bar{X})(Y_i - \bar{Y}) = -598.57$

b) $b_1 = \dfrac{-598.57}{32.857} = -18.217$

For a one dollar increase in the price per gallon of paint, the quantity sold per seven days of operation would decrease by 18.217 gallons of paint.

c) $b_0 = 138.5714 + 18.217 * 7.1429 = 268.70$

If the price of the paint were \$0 per gallon, we would expect to sell 268.7 gallons per 7 days of operation

Note! We are extrapolating the results beyond the observed data. Interpret w/ caution!

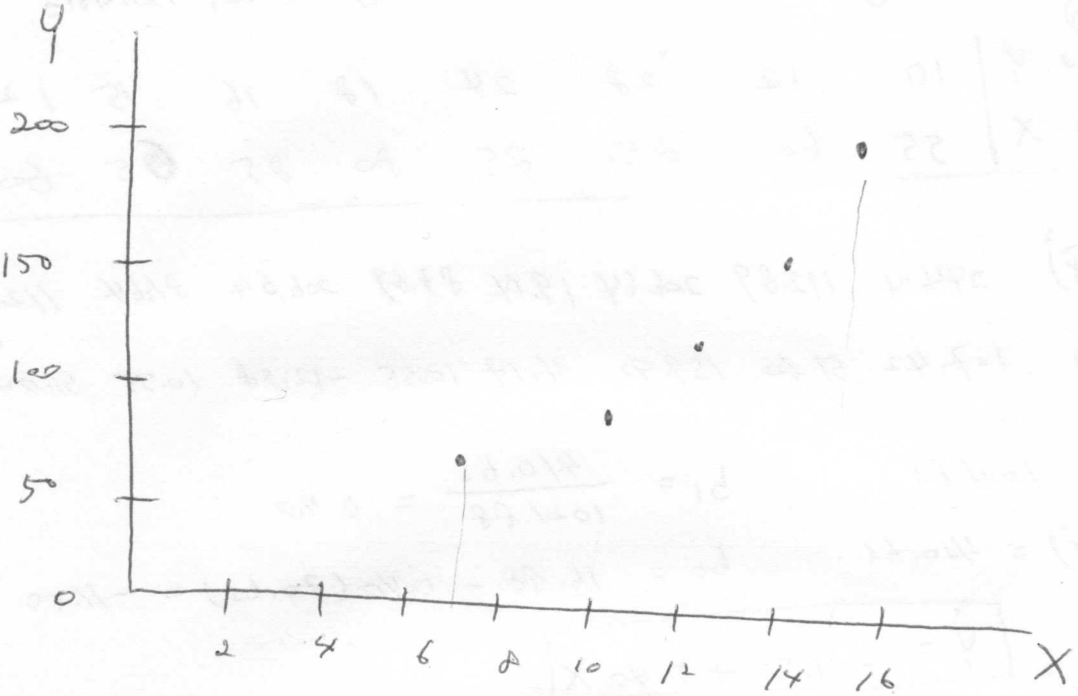
d) $X = 7$,

$\hat{Y} = 268.70 - 18.217 * 7 = 141.181$ gallons.

Q3

expenditure X	10	15	7	12	14
Sales Y	100	200	80	120	150

a)



b) It appears that advertising hav a positive effect on sales.

c) $\bar{X} = 11.60$, $\bar{Y} = 130.00$

$\Sigma (X_i - \bar{X})^2 = 41.2$

$\Sigma (X_i - \bar{X})(Y_i - \bar{Y}) = 560$

$b_1 = \frac{560}{41.2} = 13.5922$

$b_2 = 130 - 13.5922(11.60) = -27.669$

Q4. $Y = 43 + 10X$

a) $\Delta X = 8 \rightarrow \Delta Y = {}^{+}80$

b) $\Delta X = -6 \rightarrow \Delta Y = -60$

c) $\hat{Y} = 43 + 10(11) = 153$

d) $\hat{Y} = 43 + 10(29) = 333$

e) Regression models results do not "prove" that increased values of X "causes" increased values of Y.

Q5. There are insufficient data points to create an accurate regression equation. The estimate is significantly outside previous data values so any estimate would be faulty & not reliable.

Q6.

($000)								
Weekly Sales Y	10	12	28	24	18	16	15	12
Test score X	55	60	85	75	80	85	65	60

$\bar{Y} = 16.88$

$\bar{X} = 70.63$

$(X_i-\bar{X})^2$	244.14	112.89	206.64	19.14	87.89	206.64	31.64	112.89
$(X_i-\bar{X})(Y_i-\bar{Y})$	107.42	57.80	159.92	31.17	10.55	-12.58	10.55	57.80

$\Sigma(X_i-\bar{X})^2 = 1021.88$

$\Sigma(X_i-\bar{X})(Y_i-\bar{Y}) = 410.63$

$b_1 = \frac{410.63}{1021.88} = 0.40$

$b_0 = 16.88 - 0.40(70.63) = -11.50$

a) $\boxed{\hat{Y} = -11.50 + 0.40X}$

b) For one unit increase in the test score, we estimate / predict that there will be an increase of \$401.84 in weekly sales.

Q7.

$SSR = \Sigma(\hat{Y}_i - \bar{Y})^2$

$SSE = \Sigma(Y_i - \hat{Y}_i)^2$

$Se^2 = \frac{SSE}{n-2}$

$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$

$SST = \Sigma(Y_i - \bar{Y})^2$.

a) $SST = 100000$

$n = 52$ $R^2 = 0.50$

$0.50 = \frac{SSR}{100000} \Rightarrow \boxed{SSR = 50,000}$

$SST = SSR + SSE \rightarrow 100,000 = 50000 + SSE \rightarrow \boxed{SSE = 50,000}$

$\boxed{Se^2 = \frac{50000}{50} = 1000}$

b) $n=52$

$0.7 = \frac{SSR}{90000} \rightarrow \boxed{SSR = 63000.}$ $\boxed{SSE = \underset{SST}{90000} - \underset{SSR}{63000} = 27000.}$

$\boxed{Se^2 = \frac{27000}{50} = 540}$

c)

$SSR = 0.8 * 240 = 192$

$SSE = 240 - 192 = 48.$

$Se^2 = \frac{48}{50} = 0.96$

d)

$SSR = 0.3 * 200000 = 60,000$

$SSE = 200000 - 60000 = 140000$

$Se^2 = \frac{140000}{72} = 1944.44$

e)

$SSR = 0.9 * 60000 = 54000$

$SSE = 60000 - 54000 = 6000$

$Se_2 = \frac{6000}{38} = 157.89$

$\bar{X} = 180, \ \bar{Y} = 405$ (5)

Q8.

Sales (Y)	420	380	350	400	440	380	450	420
Price (X)	98	194	244	207	89	261	149	198
$(X_i-\bar{X})^2$	6724	196	4096	729	8281	6561	961	324
$(X_i-\bar{X})(Y_i-\bar{Y})$	-1230	-350	-3520	-135	-3185	-2025	-1395	270
$\hat{Y}_i$ = 479.72 − 0.415(98) =	439.04	399.19	378.43	393.79	442.78	371.38	417.87	397.53
$(\hat{Y}_i-\bar{Y})^2$	1158.67	33.77	705.81	125.62	1426.96	1130.58	165.60	55.83
$(Y_i-\bar{Y})^2$	225	625	3025	25	1225	625	2025	225

$\Sigma(X_i-\bar{X})^2 = 27872$

$\Sigma(X_i-\bar{X})(Y_i-\bar{Y}) = -11570$

$b_1 = \dfrac{-11570}{27872} = -0.415$

$b_0 = 405 - (-0.415) * 180 = 479.72$

$\boxed{\hat{Y}_i = 479.72 - 0.415X}$

$SST = \Sigma(Y_i-\bar{Y})^2 = 8000$

$SSR = \Sigma(\hat{Y}_i-\bar{Y})^2 = 4802.85 \rightarrow \boxed{R^2 = \dfrac{SSR}{SST} = \dfrac{4802.85}{8000} = 0.60}$

Q9. $\boxed{\begin{aligned} H_0 &: \beta_1 = 0 \\ H_1 &: \beta_1 > 0 \\ \alpha &= 0.05 \end{aligned}}$

a. Reject H_0 if $t_{n-2,0.05} < t_c = \dfrac{b_1-\beta_1}{S_{b_1}} = \dfrac{5}{2.1} = 2.38$

$t_{n-2,0.05} = t_{36,0.05} \doteq 1.684$

Since $t_{n-2,0.05} = 1.684 < t_c = 2.38$

Reject H_0.

b.

$b_1 - t_{36,0.025} \cdot S_{b_1} < \beta_1 < b_1 + t_{36,0.025} S_{b_1}$

$5 - 2.021 * 2.1 < \beta_1 < 5 + 2.021 * 2.1$

$0.7559 < \beta_1 < 9.2441$

6. Reject H_0 if $t_{44,0.05} < t_c = \dfrac{5.2}{2.1} = 2.476$

$t_{44,0.05} = 1.684$

Since $t_{44,0.05} < t_c$, Reject H_0

$5 - \underbrace{t_{44,0.025}}_{2.021} \cdot 2.1 < \beta_1 < 5 + 2.021 \cdot 2.1$

$0.7559 < \beta_1 < 9.2441$

For $\alpha = 0.01$, use $t_{44,0.005} \doteq 2.704$

c. Reject H_0 if $\underbrace{t_{36,0.05}}_{1.684} < t_c = \dfrac{2.7}{1.87} = 1.44$

$\rightarrow$ Fail to reject.

$2.7 - \underbrace{t_{36,0.025}}_{2.021} * 1.87 < \beta_1 < 2.7 + t_{36,0.025} * 1.87$

d. Reject Ho if $t_{27,0.05} < t_c = \frac{6.7}{1.8} = 3.722$

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$\quad\quad\quad\quad 1.703 \quad \rightarrow$ Reject Ho.

$b_1 - t_{27,0.025} S_{b_1} < \beta_1 < b_1 + t_{27,0.025} S_{b_1}$

$6.7 - 2.052 * 1.8 < \beta_1 < 6.7 + 2.052 * 1.8$

$3.0064 < \beta_1 < 10.3936$

Use $t_{27,0.005} = 2.771$
for 99% CI

Q10. a) $Se^2 = \frac{SSE}{n-2} = \frac{\Sigma(Y_i - \hat{Y}_i)^2}{n-2}$ $\quad n = 8$

$\bar{X} = 177.25$
$\bar{Y} = 405.125$

Y	418	384	343	407	432	386	444	427
X	98	194	231	207	89	255	149	195
$(X_i - \bar{X})^2$	6280.56	280.56	2889.06	885.06	7788.06	6045.06	798.05	315.06
$(X-\bar{X})(Y-\bar{Y})$	−1020.84	−353.84	−3339.22	55.78	−2371.72	−1486.97	−1098.22	388.28
$\hat{Y}_i$	434.05	399.01	385.15	394.27	437.33	376.75	415.43	398.65
$(Y_i - \hat{Y}_i)$	257.49	225.37	1807.05	162.60	28.42	85.15	815.98	803.88

$b_1 = -0.36494 \leftarrow \frac{\Sigma(X-\bar{X})(Y-\bar{Y}) = -9226.25}{\Sigma(X-\bar{X})^2 = 25281.50}$

$b_0 = 469.81 \leftarrow \bar{Y} - b_1 * \bar{X}$.

$Se^2 = \frac{SSE}{n-2} = \frac{\Sigma(Y_i - \hat{Y}_i)}{n-2} = \cancel{\frac{8736.44}{6}} = \cancel{1456.07}$

$$\boxed{Se^2 = \frac{4185.84}{6} = 697.64}$$

b) $Sb_1^2 = \frac{Se^2}{\underbrace{(n-1)Sx^2}_{=\Sigma(X_i - \bar{X})^2}} = \frac{697.64}{25281.50} = 0.028$

$Sb_1 = \sqrt{0.028} = 0.167$

c) $b_1 - t_{6,0.05} Sb_1 < \beta_1 < b_1 + t_{6,0.05} Sb_1$

$-0.36494 - 1.943 * 0.167 < \beta_1 < -0.36494 + 1.943 * 0.167$

$-0.6894 < \beta_1 < -0.040459$