

ECO240 Homework Questions for Chapter 9 (Hypothesis Tests of a Single Population)

Q1 (9.7) A random sample of  $n = 25$  is obtained from a population with variance  $\sigma^2$ , and the sample mean is computed. Test the null hypothesis  $H_0: \mu = 100$  versus the alternative hypothesis  $H_1: \mu > 100$  with  $\alpha = 0.05$ . Compute the critical value  $\bar{x}_c$  and state your decision rule for the following options:

- The population variance is  $\sigma^2 = 225$ .
- The population variance is  $\sigma^2 = 900$ .
- The population variance is  $\sigma^2 = 400$ .
- The population variance is  $\sigma^2 = 600$ .

Q2 (9.9) A random sample is obtained from a population with a variance of  $\sigma^2 = 400$ , and the sample mean is computed to be  $\bar{x}_c = 70$ . Consider the null hypothesis  $H_0: \mu = 80$  versus the alternative hypothesis  $H_1: \mu > 80$ . Compute the p-value for the following options:

- Sample size  $n = 25$
- Sample size  $n = 16$
- Sample size  $n = 44$
- Sample size  $n = 32$

Q3 (9.12) A company that receives shipments of batteries tests a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with a standard deviation of 3 hours. For one particular shipment the mean lifetime for a sample of nice batteries was 48.2 hours. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.

Q4 (9.15) Test the hypotheses  $H_0: \mu = 100$ ,  $H_1: \mu < 100$  using a random sample of  $n = 36$ , a probability of Type I error equal to 0.05, and the following sample statistics:

- $\bar{x} = 106$ ;  $s = 15$
- $\bar{x} = 104$ ;  $s = 10$
- $\bar{x} = 95$ ;  $s = 10$
- $\bar{x} = 92$ ;  $s = 18$

Q5 (9.19) A random sample of 172 marketing students was asked to rate on a scale from 1 (not important) to 5 (extremely important) health benefits as a job characteristics. The sample mean rating was 3.31, and the sample standard deviation was 0.70. Test at the 1% significance level the null hypothesis that the population mean rating is at most 3.0 against the alternative that it is bigger than 3.0.

Q6 (9.25) A statistics instructor is interested in the ability of students to assess the difficulty of a test they have taken. This test was taken by a large group of students, and the average score was 78.5. A random sample of eight students was asked to predict this average score. Their predictions were as follows:

72 83 78 65 69 77 81 71

Assuming a normal distribution, test the null hypothesis that the population mean prediction would be 78.5. Use a two-sided alternative and a 10% significance level.

Q7 (9.27) In contract negotiations a company claims that a new incentive scheme has resulted in average weekly earnings of at least \$400 for all customer service workers. A union representative takes a random sample of 15 workers and finds that their weekly earnings have an average of \$381.35 and a standard deviation of \$48.60. Assume a normal distribution.

- a. Test the company's claim
- b. If the same sample results had been obtained from a random sample of 50 employees, could the company's claim be rejected at a lower significance level than that used in part (a)?

Q8 (9.37) Consider a problem with the hypothesis test  $H_0: \mu = 5$  and  $H_1: \mu > 5$  and the following decision rule: Reject  $H_0$  if  $\bar{x} > 5 + 1.645 (0.1/\sqrt{16}) = 5.041$ . Compute the probability of Type II error and the power for the following true population means:

- a.  $\mu = 5.10$
- b.  $\mu = 5.03$
- c.  $\mu = 5.15$
- d.  $\mu = 5.07$

Q9 (9.39) A company that receives shipments of batteries tests a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with a standard deviation of 3 hours. For one particular shipment the mean lifetime for a sample of nine batteries was 48.2 hours.

- a. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.
- b. Find the power of a 10% level test when the true mean lifetime of batteries is 49 hours.

Q10 (9.56) state whether each of the following is true or false.

- a. The significance level of a test is the probability that the null hypothesis is false.
- b. A Type I error occurs when a true null hypothesis is rejected.
- c. A null hypothesis is rejected at the 0.025 level, but is not rejected at the 0.01 level. This means that the p-value of the test is between 0.01 and 0.025.
- d. The power of a test is the probability of accepting a null hypothesis that is true.
- e. If a null hypothesis is rejected against an alternative at the 5% level, then using the same data, it must be rejected against that alternative at the 1% level.
- f. If a null hypothesis is rejected against an alternative at the 1% level, then using the same data, it must be rejected against the alternative at the 5% level.
- g. The p-value of a test is the probability that the null hypothesis is true.