Eco240 HW Answers for Ch. 9 (Hypothesis Tests of a Single population)

Q1. $n = 25$, $\alpha = 0.05$.

$H_0: \mu = 100$

$H_1: \mu > 100$

a. $\sigma^2 = 225$. Reject Ho if $\bar{X} > \bar{X}_c = \mu_0 + \underset{\overset{\parallel}{z_{0.05}}}{z_\alpha} \frac{\sigma}{\sqrt{n}} = 100 + 1.645\sqrt{\frac{225}{25}}$

$= 104.935$

b. $\sigma^2 = 900$ Reject Ho if $\bar{X} > 100 + 1.645\sqrt{\frac{900}{25}} = 109.87$

c. $\sigma^2 = 400$ 〃 $\bar{X} > 100 + 1.645\sqrt{\frac{400}{25}} = 106.58$

d. $\sigma^2 = 600$ 〃 $\bar{X} > 100 + 1.645\sqrt{\frac{600}{25}} = 108.0588$.

Q2. $\sigma^2 = 400$

$H_0: \mu = 80$

$H_1:$ ~~$\mu > 80$~~ $\mu < 80$

$\bar{X}_0 = 70$ (Observed $\bar{X}$.)

Not $\bar{X}_c$ ← typo in the question

sorry!

a. $n = 25$. $z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{70 - 80}{20/\sqrt{25}} = -2.50$

p-value $= P(z_p < -2.50) = 0.0062$.

b. $n = 16$. $z = \frac{70 - 80}{20/\sqrt{16}} = -2.00$

p-value $= P(z_p < -2.00) = 1 - P(z < 2.00)$

$= 0.0228$

c. $n = 44$. $z = \frac{70 - 80}{20/\sqrt{44}} = -3.32$. $p = P(z_p < -3.32) = 1 - P(z < 3.32)$

$= 0.0004$

d. $n = 32$. $z = \frac{70 - 80}{20/\sqrt{32}} = -2.83$ $p = P(z_p < -2.83) = 1 - P(z < 2.83)$

$= 0.0023$

Q3. $\sigma = 3$.

$\bar{X} = 48.2$

$\alpha = 0.1$

$$\boxed{\begin{array}{l} H_0: \mu \geq 50 \\ H_1: \mu < 50 \end{array}}$$

Reject Ho if $\bar{X} < \bar{X}_c = \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}}$

$= 50 - 1.28 \frac{3}{\sqrt{9}} = 48.72$

since $\bar{X} = 48.2$,

$\bar{X} = 48.2 < \bar{X}_c = 48.72$ applies, we can reject Ho.

Q4.

$H_0: \mu = 100$
$H_1: \mu < 100$
$n = 36$
$\alpha = 0.05$

a. $\bar{X} = 106,\ S = 15$

Reject H_0 if $\bar{X} < \bar{X}_c = \mu_0 - t_{n-1,\alpha}\frac{S}{\sqrt{n}}$

$= 100 - \underbrace{t_{35,0.05}}_{\underset{1.697}{\shortparallel}}\frac{15}{\sqrt{36}} = 95.7575$

since $\bar{X} = 106 > \bar{X}_c = 95.76$

→ we ~~fail to~~ fail to reject H_0

b. $\bar{X} = 104,\ S = 10$.

Reject H_0 if $\bar{X} < \bar{X}_c = 100 - 1.697\frac{10}{\sqrt{36}} = 97.17$.

since $\bar{X} = 104 > \bar{X}_c = 97.17$, we fail to reject H_0

c. $\bar{X} = 95,\ S = 10$.

Reject H_0 if $\bar{X} < \bar{X}_c = 100 - 1.697\frac{10}{\sqrt{36}} = 97.17$.

since $\bar{X} = 95 < \bar{X}_c = 97.17$, we reject H_0

d. $\bar{X} = 92,\ S = 18$.

Reject H_0 if $\bar{X} < \bar{X}_c = 100 - 1.697 \cdot \frac{18}{\sqrt{36}} = 94.909$

since $\bar{X} = 92 < \bar{X}_c = 94.909$, we reject H_0

Q5. $n = 172$
$\bar{X} = 3.31$
$S = 0.7$
$\alpha = 0.01$

$H_0: \mu \le 3.0$
$H_1: \mu > 3.0$

Reject H_0 if $\bar{X} > \bar{X}_c = \mu_0 + t_{n-1,\alpha}\frac{S}{\sqrt{n}}$

$= 3 + \underbrace{t_{171,0.01}}_{\underset{2.326}{\shortparallel}} * \frac{0.7}{\sqrt{172}}$

$= 3.124$

since $\bar{X} = 3.31 > \bar{X}_c = 3.124$

We reject H_0.

Q6. $\mu = 78.5$
$\bar{X} = 74.5$

$H_0: \mu = 78.5$
$H_1: \mu \ne 78.5$

$\alpha = 0.1$
$n = 8$

$$S = \sqrt{\frac{(72-74.5)^2 + (83-74.5)^2 + (78-74.5)^2 + (65-74.5)^2 + (69-74.5)^2 + (77-74.5)^2 + (81-74.5)^2 + (71-74.5)^2}{7}}$$

$= \sqrt{\frac{272}{7}} = 6.23$

Reject H_0 if $\bar{X} < \bar{X}_c = \mu_0 - t_{7,0.05}\frac{6.23}{\sqrt{8}} = 74.33$ or $\bar{X} > \bar{X}_c = 78.5 + 1.895\frac{6.23}{\sqrt{8}} = 82.67$

$\boxed{\text{Since } \bar{X} = 74.5, \text{ we can't reject } H_0}$

Q7.

$n = 15$ $\quad H_0: \mu \geq 400$

$\bar{X} = 381.35$ $\quad H_1: \mu < 400$

$s = 48.60$

a. Reject H_0 if $\bar{X} < \bar{X}_c = 400 - t_{14,\alpha} \frac{48.60}{\sqrt{15}}$

→ since we are not given α, let's approach from different angle.

$$t = \frac{381.35 - 400}{48.60/\sqrt{15}} = -1.486$$

$$p\text{-value} = P(t_p < -1.486)$$
$$= P(t_p > 1.486)$$

between 0.1 and 0.05. or 5% − 10%

We reject H_0 if α is greater than exact p-value which is between 5 & 10%

if $\alpha = 0.05$,

$$\bar{X}_c = 400 - 1.761 \frac{48.60}{\sqrt{15}} = 377.9$$

$\bar{X} = 381.35 > \bar{X}_c$ 377.9 → Fail to reject H_0

if $\alpha = 0.10$

$$\bar{X}_c = 400 - 1.345 \frac{48.60}{\sqrt{15}} = 383.12$$

Since $\bar{X} = 381.35 < \bar{X}_c = 383.12$ → Reject H_0

b. $n = 50$.

$$t = \frac{381.35 - 400}{48.60/\sqrt{50}} = -2.713$$

$p\text{-value} = P(t_p < -2.713) = P(t_p > 2.713)$ = between 0.01 and 0.005 (0.5 − 1 %)

Therefore, the claim can be rejected at lower significance level

Q8. $H_0: \mu = 5$ $\quad \bar{X}_c = 5.041$

$H_1: \mu > 5$

Compute Type II error : β.

$$\beta = P\left(Z < \frac{\bar{X}_c - \mu_1}{\sigma/\sqrt{n}}\right)$$

a. $\mu_1 = 5.10$

$$\beta = P\left(Z < \frac{5.041 - 5.10}{0.1/\sqrt{16}}\right) = P(Z < -2.36)$$

Type II error

$$= 1 - \underbrace{P(Z < 2.36)}_{= 0.9909} = 1 - 0.9909 = 0.0091$$

$1 - \beta =$ (power) 0.9909

b. $\mu_1 = 5.03$

$$\beta = P\left(Z < \frac{5.041 - 5.03}{0.1/\sqrt{16}}\right) = P(Z < 0.44) = 0.6700$$

$1 - \beta = 0.3300$

c. $\mu_1 = 5.15$

$$\beta = P\left(Z < \frac{5.041 - 5.15}{0.1/\sqrt{16}}\right) = P(Z \le -4.36) = 0.0000$$

$1 - \beta = 1.$

d. $\mu_1 = 5.07$

$$\beta = P\left(Z < \frac{5.041 - 5.07}{0.1/\sqrt{16}}\right) = P(Z < -1.16) = 0.3770$$

$1 - \beta = 0.6230$

Q9.

$H_0: \mu \geq 50$
$H_1: \mu < 50$
$\alpha = 0.1$
$\sigma = 3$
$\bar{X} = 48.2$
$n = 9$

a) Reject Ho if $\bar{X} < \bar{X}_c = \mu_0 - z_{0.1}\frac{\sigma}{\sqrt{n}}$

$= 50 - 1.28\frac{3}{\sqrt{9}}$

$= 48.72$

Since $\bar{X} = 48.2 < \bar{X}_c = 48.72$,
we reject Ho

b. $\mu_1 = 49$.

$$\beta = P\left(Z > \frac{\overset{\bar{X}_c}{48.72} - \overset{\mu_1}{49}}{3/\sqrt{9}}\right) = P(Z > -0.28) = P(Z < 0.28)$$

$= 0.6103$

Power of the test.

$\boxed{1 - \beta = 0.3897}$

Q10. a) False. The significance level is the probability of making a Type I error - falsely rejecting the null hypothesis when in fact the null is true.

b) True

c) True

d) False. The power of the test is the ability of the test to correctly reject a false null hypothesis

d.) False. The rejection region is farther away from the Hypothesized value at the 1% level than it is at the 5% level. Therefore, it is still possible to reject at the 5% level but not at the 1% level.

f) True

g) False. The p-value tells the strength of the evidence against the null hypothesis.