EC0240 HW2 Ch7 Ansrers

Q1. (7.1) a. $\bar{X} = \dfrac{6+8+7+10+3+5+9+8}{8} = 7.0$

b. $\sigma^2_{\bar{X}} = \dfrac{\sigma^2}{n}$ → Unbiased estimator of $\sigma^2 = S^2$.

$$Var(\bar{X}) = \frac{S^2}{n} = \left(\underbrace{\frac{(3-7)^2+(5-7)^2+(6-7)^2+(7-7)^2+2(8-7)^2+(9-7)^2+(10-7)^2}{8-1}}_{=S^2}\right) \Big/ 8$$

$$= \frac{(2.268)^2}{8} = 0.64298.$$

Q2. (7.6) a. $E(X) = \frac{1}{2}\mu + \frac{1}{2}\mu = \mu$

$E(Y) = \frac{1}{4}\mu + \frac{3}{4}\mu = \mu$

$E(Z) = \frac{1}{3}\mu + \frac{2}{3}\mu = \mu$

b. $\boxed{Var(X) = \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \frac{1}{2}\sigma^2}$ Smallest Variance The most efficient.

$Var(Y) = \frac{1}{16}\sigma^2 + \frac{9}{16}\sigma^2 = \frac{10}{16}\sigma^2$

$Var(Z) = \frac{1}{9}\sigma^2 + \frac{4}{9}\sigma^2 = \frac{5}{9}\sigma^2$

c. R.E

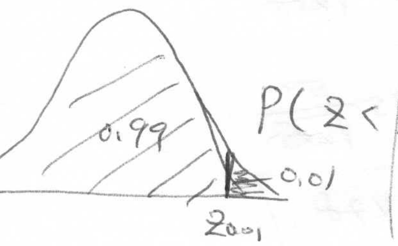
X wrt Y $\dfrac{Var(Y)}{Var(X)} = \dfrac{\frac{10}{16}\sigma^2}{\frac{1}{2}\sigma^2} = \dfrac{20}{16} = \dfrac{5}{4} = 1.25$

X wrt Z. $\dfrac{Var(Z)}{Var(X)} = \dfrac{\frac{5}{9}\sigma^2}{\frac{1}{2}\sigma^2} = \dfrac{10}{9} = 1.11.$

Q3 (7.10) a. $\alpha = 0.02$, $n = 64$, $\sigma^2 = 144$ since σ^2 is given (known) use Z.

$M.E = Z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$

$= Z_{0.01} \cdot \dfrac{12}{8}$ → $Z_{0.01} = 2.33$



$P(Z < Z_{0.01}) = 0.99$ (look for 0.49 inside the table.)

$= 2.33 \times \dfrac{3}{2} = 3.495.$

b. $\alpha = 0.01$, $n = 120$, $\sigma = 100$.

$$ME = Z_{0.005} \frac{100}{\sqrt{120}}$$

$P(Z < Z_{0.005}) = 0.995$. look for 0.495 inside the table

$\rightarrow Z_{0.005} = 2.57$ or 2.58.

$$ME = 2.57 * \frac{100}{\sqrt{120}} = 23.46$$

or

$$= 2.58 * \frac{100}{\sqrt{120}} = 23.552.$$

Q4. (7.12)

a. $LCL = \bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$= 50 - Z_{0.025} \frac{40}{\sqrt{64}}$$

$$= 50 - 1.96 \frac{40}{8} = \boxed{40.2}$$

$UCL = \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$= 50 + 1.96 \frac{40}{8}$$

$$= \boxed{59.8.}$$

$P(Z < Z_{0.025}) = 0.975$.

look for 0.475 inside the table.

b. $LCL = 85 - Z_{0.005} \frac{20}{\sqrt{225}}$

$P(Z < Z_{0.005}) = 0.995$.

$Z_{0.005} = 2.58$.

$$= 85 - 2.58 \frac{20}{\sqrt{225}}$$

$$= \boxed{81.56}$$

$UCL = 85 + 2.58 \frac{20}{\sqrt{225}}$

$$= \boxed{88.44}$$

c. $LCL = 510 - Z_{0.05} \frac{50}{\sqrt{485}}$

$P(Z < Z_{0.05}) = 0.95$.

$Z_{0.05} = 1.645$.

$$= 510 - 1.645 \frac{50}{\sqrt{485}}$$

$$= \boxed{506.2652}$$

$UCL = 510 + Z_{0.05} \frac{50}{\sqrt{485}}$.

$$= 510 + 1.645 \frac{50}{\sqrt{485}}$$

$$= \boxed{513.73478.}$$

Q5. (7.13) $\sigma = 32.4$, $n = 9$, $\bar{x} = 187.9$

a. $1-\alpha = 80\%$ $\quad \alpha = 0.2$, $\alpha/2 = 0.1$

$$187.9 - Z_{0.1}\frac{32.4}{3} < \mu < 187.9 + Z_{0.1}\frac{32.4}{3}$$

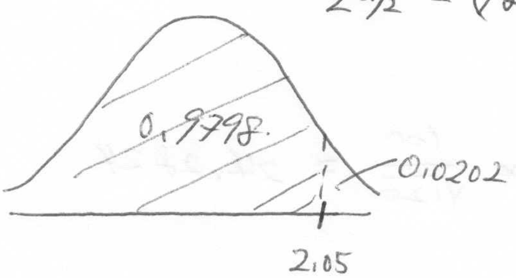
$$P(Z < Z_{0.1}) = \underset{\text{look for } 0.4}{0.9} \rightarrow Z_{0.1} = 1.28$$

$$187.9 - 1.28\frac{32.4}{3} < \mu < 187.9 + 1.28\frac{32.4}{3}$$

$$\boxed{174.076 < \mu < 201.724}$$

b. $$187.9 - Z_{\alpha/2}\frac{32.4}{3} = 165.8$$

$$Z_{\alpha/2} = (187.9 - 165.8) * \left(\frac{3}{32.4}\right) = 2.05$$



$$\frac{\alpha}{2} = [1 - \underbrace{F_Z(2.05)}_{0.9798}] = 0.0202$$

from the table

Confidence level $= 1-\alpha$

$$1 - 2*\left(\frac{\alpha}{2}\right) = 1 - 2(0.0202) = 0.9596$$

$\rightarrow \boxed{95.96\%}$ confidence level

Q6. (7.15) $\sigma = 0.45$, $n = 25$, $\bar{x} = 2.90$

a. $\alpha = 0.05$, $\alpha/2 = 0.025$

$$2.9 - Z_{0.025}\frac{0.45}{5} < \mu < 2.9 + Z_{0.025}\frac{0.45}{5}$$

$$P(Z < Z_{0.025}) = 0.975 \rightarrow Z_{0.025} = 1.96$$

$$2.9 - 1.96\frac{0.45}{5} < \mu < 2.9 + 1.96\frac{0.45}{5}$$

$$2.7236 < \mu < 3.0764$$

b. $$2.9 - Z_{\alpha/2}\frac{0.45}{5} = 2.81$$

$$Z_{\alpha/2} = (2.9 - 2.81) * \frac{5}{0.45} = 1$$

$$\frac{\alpha}{2} = [1 - \underbrace{F_Z(1)}_{0.8413}] = 0.1587$$

$CL = 1-\alpha$

$$1 - 2\left(\frac{\alpha}{2}\right) = 1 - 2(0.1587) = 0.6826$$

$$\boxed{CL = 68.26\%}$$

Q7 (7.17)

a. $n = 20, \alpha = 0.1$

$t_{19, 0.05} = 1.729$

b. $n = 7, \alpha = 0.02$

$t_{6, 0.01} =$ [redacted] 3.143.

c. $n = 16, \alpha = 0.05$

$t_{15, 0.025} = 2.131$.

d. $n = 23, \alpha = 0.01$

$t_{22, 0.005} = 2.819$

Q8. (7.21)

a. $\alpha = 0.02, n = 64, s^2 = 144$

$$M.E. = \underbrace{t_{63, 0.01}}_{= 2.390} \cdot \frac{s}{\sqrt{n}} = 2.39 * \frac{12}{8} = 3.585.$$

* approximate w/ $\nu = 60$ value.

b. $\alpha = 0.01, n = 120, s = 100$

$$ME = \underbrace{t_{119, 0.005}}_{= 2.660} \frac{s}{\sqrt{n}} = 2.660 * \frac{100}{\sqrt{120}} = 24.2824$$

* approximate w/ $\nu = 60$ value.

c. $\alpha = 0.05, n = 200, s = 40.$

$$ME = \underbrace{t_{199, 0.025}}_{= 2} \frac{40}{\sqrt{200}} = 5.657.$$

* approximate w/ $\nu = 60$ valu

Q9. (7.26) $n=7$, $X = 79, 73, 68, 77, 86, 71, 69$

$\alpha = 0.05$, $\alpha/2 = 0.025$

① Compute S (sample standard error)

$\bar{X} = 74.7143$

$$S = \sqrt{\frac{(68-74.7143)^2+(71-74.7143)^2+(69-74.7143)^2+(73-74.7143)^2+(77-74.7143)^2+(79-74.7143)^2+(86-74.7143)^2}{7-1}}$$

$= 6.3957.$

② $ME = \underbrace{t_{6,0.025}}_{=\,2.447} \times \frac{6.3957}{\sqrt{7}} = \boxed{29.8934}$

Q10. (7.27) $n=10$, $X = 6, 11, 12, 15, 16, 17, 18, 19, 20, 25.$

a. $\bar{X} = 15.90.$

$$S = \sqrt{\frac{(6-15.9)^2+(11-15.9)^2+(12-15.9)^2+(15-15.9)^2+(16-15.9)^2+(17-15.9)^2+(18-15.9)^2+(19-15.9)^2+(20-15.9)^2+(25-15.9)^2}{10-1}}$$

$= 5.30$

$\alpha = 0.01$, $\alpha/2 = 0.005.$

$$\bar{X} - \underbrace{t_{9,0.005}}_{=} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{9,0.005}\frac{S}{\sqrt{n}}$$

$$15.90 - 3.25(5.30/\sqrt{10}) < \mu < 15.90 + 3.25(5.30/\sqrt{10})$$

$$\boxed{10.453 < \mu < 21.347}$$

b. 90% Confidence interval → narrower than 99% C.I.

Since t value is smaller for a 90% CI.

Q11. (7.44) $n=5$, $X = 8, 9, 10, 12, 16$.

a. $\alpha = 0.1$, $\alpha/2 = 0.05$

$$\frac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}$$

① Find S

$\bar{X} = 11$

$$S = \sqrt{\frac{(8-11)^2+(9-11)^2+(10-11)^2+(12-11)^2+(16-11)^2}{4}} = \sqrt{10} = 3.16$$

② Find $\chi^2_{n-1,\alpha/2}$, $\chi^2_{n-1,1-\frac{\alpha}{2}}$

$\chi^2_{4,0.05} = 9.49$, $\chi^2_{4,0.95} = 0.711$

③ compute C.I.

$$\frac{4\,\cancel{(3.16)^2}^{\,*10}}{9.49} < \sigma^2 < \frac{4\,\cancel{(3.16)^2}^{\,*10}}{0.711}$$

$$\boxed{4.21496 < \sigma^2 < 56.259}$$

b. $\alpha = 0.05$, $\alpha/2 = 0.025$.

$\chi^2_{4,0.025} = 11.14$

$\chi^2_{4,0.975} = 0.484$

$$\frac{40}{11.14} < \sigma^2 < \frac{40}{0.484} \Rightarrow \boxed{3.59066 < \sigma^2 < 82.6446}$$

Q12. (7.47) $n = 20$, $S^2 = 6.62$, $\alpha = 0.05$, $\alpha/2 = 0.025$

$\chi^2_{19,0.025} = 32.85$

$\chi^2_{19,1-0.025} = 8.91$
$= 0.975$

$$\frac{19 * 6.62}{32.85} < \sigma^2 < \frac{19 * 6.62}{8.91}$$

$$\boxed{3.8289 < \sigma^2 < 14.1167}$$

Q13. (7.49) $n=15$, $s=2.36$ ⑦

a. $\alpha=0.05$, $\alpha/2=0.025$

$$\frac{14(2.36)^2}{\chi^2_{14,0.025}} < \sigma^2 < \frac{14(2.36)^2}{\chi^2_{14,0.975}}$$

$\chi^2_{14,0.025} = 26.12$

$\chi^2_{14,0.975} = 5.63$

$$\boxed{2.9852 < \sigma^2 < 13.8498}$$

b. Wider since chi-square statistic is larger for a 99 CI than for a 95% CI.

Q14. (7.57) $N=820$, $n=61$, $\bar{X}=127.43$, $s=43.27$

a. $\bar{X}=127.43$

since $\frac{61}{820}=0.074$

n is 7.4% of N. → consider large sample case

b. $$\sigma^2_{\bar{X}} = \left(\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\right)^2$$

$$= \left(\frac{43.27}{\sqrt{61}}\sqrt{\frac{820-61}{820-1}}\right)^2 = 28.44$$

c. $\alpha=0.1$, $\alpha/2=0.05$

$$127.43 - \underbrace{t_{60,0.05}}_{=1.671}\cdot\sqrt{28.44} < \mu < 127.43 + \underbrace{t_{60,0.05}}_{=1.671}\sqrt{28.44}$$

$$\boxed{118.52 < \mu < 136.34}$$

d. $127.43 - t_{60,\alpha/2}\sqrt{28.44} = 117.43$

$t_{60,\alpha/2} = (127.43-117.43)/\sqrt{28.44}$

$= 1.875$

$\alpha/2$ is between 0.05 & 0.025

α is between 0.1 & 0.05

$1-\alpha$ is between 0.9 & 0.95

CL is between 90% & 95%

* In exams, I'll specify when to use CLT.

Or, since $n>25$, use Central Limit Theorem

$127.43 - z_{\alpha/2}\sqrt{28.44} = 117.43$

$z_{\alpha/2} = 1.8~~75~~ 85$

$P(Z<z_{\alpha/2}) = 0.9699$

$\alpha/2 = 1-0.9699 = 0.0301$

$1-\alpha = 1-2(\frac{\alpha}{2})$

$= 1-0.0602$

$= 0.9398$

CL = 93.98%

Q ~~15~~ 15. (7.58) $N = 125,\ n = 41,\ \bar{X} = 7.28,\ S = 5.22$ (8)

a. $\alpha = 0.01,\ \alpha/2 = 0.005$

Since n is 32.8% of N, use "large sample case"

$$7.28 - \underbrace{t_{40,0.005}}_{=2.660} \frac{5.22}{\sqrt{41}}\sqrt{\frac{125-41}{125-1}} < \mu < 7.28 + t_{40,0.005}\frac{5.22}{\sqrt{41}}\sqrt{\frac{125-41}{125-1}}$$

1.819

$$\boxed{5.461 < \mu < 9.099}$$

Q16. (7.59) a. $N, S^2, n\uparrow \rightarrow CI(95\%)\uparrow$? False

$$\bar{X} - \underbrace{t_{n-1,\alpha/2}}\cdot\underbrace{\frac{S}{\sqrt{n}}} < \mu < \bar{X} + t_{n-1,\alpha/2}\cdot\frac{S}{\sqrt{n}}$$

As $n\uparrow \rightarrow t_{n-1,\alpha/2} \downarrow$

$\searrow \frac{S}{\sqrt{n\uparrow}} \downarrow$ $\rightarrow$ C.I. becomes narrower

b. $N, n, S^2\uparrow \rightarrow CI(95\%)\uparrow$? True

$$\bar{X} - t_{n-1,\alpha/2}\cdot\frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{n-1,\alpha/2}\frac{S}{\sqrt{n}}$$

As $S^2\uparrow \rightarrow S\uparrow \rightarrow \frac{S}{\sqrt{n}}\uparrow \rightarrow ME\uparrow \rightarrow CI$ wider.

c. $n, S^2, N\uparrow \rightarrow CI(95\%)\uparrow$? True

compare two cases : ① small sample case w/ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

② large sample case w/ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

$$\sqrt{\frac{N-n}{N-1}} < 1$$

$$\underset{(\text{small})}{\sigma_{\bar{X}}} = \frac{\sigma}{\sqrt{n}} > \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \underset{(\text{large sample})}{\sigma_{\bar{x}}}$$

As $N\uparrow$, move from case ② to ① $\rightarrow$ ① $\bar{X} - \underbrace{t_{v,\alpha/2}\cdot\frac{S}{\sqrt{n}}}_{\text{larger than ②}} < \mu < \bar{X} + t_{v,\alpha/2}\frac{S}{\sqrt{n}}$ $\rightarrow$ wider

② $\bar{X} - \underbrace{t_{v,\alpha/2}\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}_{\text{smaller than ①}} < \mu < \bar{X} + t_{v,\alpha/2}\cdot\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$ $\rightarrow$ narrower

d. N, n, S^2, $CI_{(95\%)} > CI_{(90\%)}$? True.

$$\bar{X} - \underbrace{t_{\nu,\alpha/2}}\frac{S}{\sqrt{n}} < \mu < \bar{X} + \underbrace{t_{\nu,\alpha/2}}\frac{S}{\sqrt{n}}$$

$$\boxed{\begin{matrix} t_{n-1,0.025} \\ \vee \\ t_{n-1,0.05} \end{matrix}}$$

$\bar{X} \pm t_{n-1,0.025}\frac{S}{\sqrt{n}}$ is wider than

$\bar{X} \pm t_{n-1,0.05}\frac{S}{\sqrt{n}}$

Q 17. (7.66) $n = 16$, $\bar{X} = 150$, $S = 12$. $\alpha = 0.05$, $\alpha/2 = 0.025$.

$$150 - t_{15,0.025}\frac{12}{\sqrt{16}} < \mu < 150 + t_{15,0.025}\frac{12}{\sqrt{16}}$$

$t_{15,0.025} = 2.131$

$$\boxed{143.607 < \mu < 156.393}$$

Q18. (7.71) $n = 457$. $\bar{X} = 3.59$, $S = 1.045$

$$\bar{X} - t_{456,\alpha/2} \cdot \frac{1.045}{\sqrt{457}} = 3.49$$

$$t_{456,\alpha/2} = (3.59 - 3.49) * \frac{\sqrt{457}}{1.045}$$

$$= 2.05$$

$\frac{\alpha}{2}$ is between 0.025 & 0.01

α is between 0.05 & 0.02

$1-\alpha$ " 0.95 & 0.98

CL " 95% & 98%

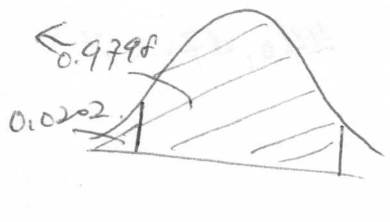
or, use Central Limit Theorem since $n > 25$,

$Z_{\alpha/2} = 2.05$

$P(Z < Z_{\alpha/2}) = 0.9798$

$CL = 0.9798 - (1 - 0.9798)$

$= 0.9596$ or 95.96%



Q19 (7.85) $N=90$, $n=10$

$X = 59, 62, 63, 71, 72, \overset{75}{\downarrow} 81, 8\cancel{4}, 87, 93$

a. $\alpha = 0.1$, $\alpha/2 = 0.05$

since ~~n~~ n is 11 % of N, use lage sample case

$$\bar{X} - t_{9,0.05} \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} < \mu < \bar{X} + t_{89,0.05} \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

① compute S:

$\bar{X} = 74.7$

$$S = \sqrt{\frac{(59-74.7)^2 + (62-74.7)^2 + (63-74.7)^2 + \cdots + (93-74.7)^2}{10-1}}$$

$= 11.44$

$$\sigma_{\bar{x}} = \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{11.44}{\sqrt{10}} \sqrt{\frac{90-10}{90-1}} = 3.42985$$

$t_{9,0.05} = \cancel{1.671}$ 1.833

~~approximate w/ n-1~~

$74.7 - \overset{1.833}{\cancel{1.671}} * 3.43 < \mu < 74.7 + \overset{1.833}{\cancel{1.671}} * 3.43$

$$\boxed{68.41 < \mu < 80.997}$$

b. 95% CI is wider since $t_{9,0.05} < t_{9,0.025}$

Q20 (7.86) $N = 272$, $n = 50$, $\bar{X} = 492.36$, $S = 149.92$.

Since n is 18% of N, use lage sample case.

a. $492.36 \pm t_{49,0.005} \cdot \frac{149.92}{\sqrt{50}} \sqrt{\frac{272-50}{272-1}}$

$t_{49,0.005} = \cancel{2.660}$ 2.704

approximate w/ $n-1 = 40$

$440.47 < \mu < 544.25$