ECO240 Statistics II, Homework Questions 2, Chapter 7: Confidence Interval Estimation

Q1. (7.1) The following data is from a random sample: 6 8 7 10 3 5 9 8.

a. Find a point estimate of the population mean that is unbiased and efficient.
b. Use an unbiased estimation procedure to find a point estimate of the variance of the sample mean.

Q2. (7.6) Suppose that x1 and x2 are random samples of observations from a population with mean μ and variance s^2. Consider the following three point estimators, X, Y, Z, of μ.

$$X = \frac{1}{2}x_1 + \frac{1}{2}x_2, \quad Y = \frac{1}{4}x_1 + \frac{3}{4}x_2, \quad Z = \frac{1}{3}x_1 + \frac{2}{3}x_2$$

a. Show that all three estimators are unbiased.
b. Which of the estimators is the most efficient?
c. Find the relative efficiency of X with respect to each of the other two estimators.

Q3. (7.10) Calculate the margin of error to estimate the population mean for the following:

a. 98% confidence level; n = 64, σ^2 =144.
b. 99% confidence level, n = 120, σ =100.

Q4. (7.12) Calculate the LCL and UCL for each of the following:

a. $\bar{x}$ = 50, n = 64, σ = 40, α = 0.05
b. $\bar{x}$ = 85, n = 225, σ^2 =400, α = 0.01
c. $\bar{x}$ = 510, n = 485, σ = 50, α = 0.10

Q5. (7.13) A personnel manager has found that historically the scores on aptitude tests given to applicants for entry-level positions follow a normal distribution with a standard deviation of 32.4 points. A random sample of nine test scores from the current group of applicants has a mean score of 187.9 points.

a. Find an 80% confidence interval for the population mean score of the current group of applicants.
b. Based on these sample results, a statistician found for the population mean a confidence interval extending from 165.8 to 210.0 points. Find the confidence level of this interval.

Q6. (7.15) A college admissions officer for an MBA program has determined that historically applicants have undergraduate grade point averages that are normally distributed with standard deviation 0.45. From a random sample of 25 applications from the current year, the sample mean grade point average is 2.90.

a. Find a 95% confidence interval for the population mean.
b. Based on these sample results, a statistician computes for the population mean a confidence interval extending from 2.81 to 2.99. Find the confidence level associated with this interval.

Q7. (7.17) Find the reliability factor, $t_{v,\alpha/2}$, to estimate population mean for the following:

a. n = 20, 90% confidence level

b. n = 7, 98% confidence level
c. n = 16, 95% confidence level
d. n = 23, 99% confidence level

Q8. (7.21) Calculate the margin of error to estimate the population mean for each of the following:

a. 98% confidence level, n = 64, s^2 = 144.
b. 99% confidence level, n = 120, s = 100.
c. 95% confidence level, n = 200, s = 40.

Q9. (7.26) There is concern about the speed of automobiles travelling over a particular stretch of highway. For a random sample of seven automobiles radar indicated the following speeds, in miles per hour: 79 73 68 77 86 71 69
Assuming a normal population distribution, find the margin of error of a 95% confidence interval for the mean speed of all automobiles traveling over this stretch of highway.

Q10. (7.27) A clinic offers a weight-loss program. A review of its records found the following amounts of weight loss, in pounds, for a random sample of 10 of its clients at the conclusion of the program: 18 25 6 11 15 20 16 19 12 17.

a. Find a 99% confidence interval for the population mean.
b. Without doing the calculations, explain whether a 90% confidence interval for the population mean would be wider than, narrower than, or the same as that found in part (a).

Q11.(7.44) Consider the following random sample from a normal population: 12 16 8 10 9.

a. Find the 90% confidence interval for population variance.
b. Find the 95% confidence interval for the population variance.

Q12. (7.47) The quality control manager of a chemical company randomly sampled twenty 100-pound bags of fertilizer to estimate the variance in the pounds of impurities. The sample variance was found to be 6.62. Find a 95% confidence interval for the population variance in the pounds of impurities.

Q13. (7.49) A manufacturer is concerned about the variability of the levels of impurity contained in consignments of raw material from a supplier. A random sample of 15 consignments showed a standard deviation of 2.36 in the concentration of impurity levels. Assume normality:

a. Find a 95% confidence interval for the population variance.
b. Would a 99% confidence interval for this variance be wider or narrower than that found in part (a)?

Q14. (7.57) An auditor, examining a total of 820 accounts receivable of a corporation, took a random sample of 61 of them. The sample mean was $ 127.43, and the sample standard deviation was $ 43.27.

a. Using an unbiased estimation procedure, find an estimate of the population mean.

b. Using an unbiased estimation procedure, find an estimate of the variance of the sample mean.
c. Find a 90% confidence interval for the population mean.
d. A statistician found, for the population mean, a confidence interval running from $ 117.43 to $137.43. What is the probability content of this interval?

Q15. (7.58) On a particular day of consumer advice bureau received 125 calls. For a random sample of 41 of these calls, it was found that mean time taken in providing the requested advice was 7.28 minutes, and the sample standard deviation was 5.32 minutes. Find a 99% confidence interval for the mean time taken per call.

Q16. (7.59) State whether each of the following statement is true or false:

a. For a given number of population members and a given sample variance, the larger the number of sample members, the wider the 95% confidence interval for the population mean.
b. For a given number of population members and a given number of sample members, the larger the sample variance, the wider the 95% confidence interval for the population mean.
c. For a given number of sample members and a given sample variance, the larger the number of population members, the wider the 95% confidence interval for the population mean. Justify your answer.
d. For a given number of population members, a given number of sample members, and a given sample variance, a 95% confidence interval for the population mean is wider than a 90% confidence interval for the population mean.

Q17. (7.66) Suppose that the owner of an Indonesian fish restaurant wants to estimate the mean number of lobsters that are sold during a typical weekday. Checking the sales records for a random sample of 16 days finds that the mean number of lobster sold is 150 per day and the sample standard deviation is 12 lobsters. With 95 % confidence estimate the number of lobsters that the restaurant should stock daily based on the available data.

Q18. (7.71) A marketing research assistant for a veterinary hospital surveyed a random sample of 457 pet owners. Respondents were asked to indicate the number of times that they visit their veterinarian each year. The sample mean response was 3.59 and the sample standard deviation was 1.045. Based on these results a confidence interval from 3.49 to 3.69 was calculated for the population mean. Find the probability content for this interval.

Q19. (7.85) A test was taken by 90 students. A random sample of 10 scores found the following results: 93 71 62 75 81 63 87 59 84 72

a. Find a 90% confidence interval for the population mean score.
b. Without doing the calculations, state whether a 95% confidence interval for the population mean would be wider or narrower than the interval found in part (a).