

ECO240 Statistics II Homework Questions for Chapter 6 [Distribution of Sample Statistics]

Note: Question numbers are from 7th edition.

Q1 (6.1). Suppose that you toss a pair of dice and write down the value of the faces from each die.

- a. What is the population distribution for one die?
- b. Determine the sampling distribution of the sample means obtained by tossing two dice.

Q2 (6.6). Given a population with a mean of 100 and a variance of 900, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 30$ is obtained. Suppose $N = 1000$.

- a. What are the mean and variance of the sampling distribution for the sample means?
- b. What is the probability that $\bar{x} > 109$?
- c. What is the probability that $96 \leq \bar{x} \leq 110$?
- d. What is the probability that $\bar{x} \leq 107$?

Q3 (6.8). Consider the population with mean 400 and variance 1600. A random sample of size $n = 35$ is obtained. Suppose $N = 1000$.

- a. What are the mean and variance of the sampling distribution for the sample means?
- b. What is the probability that $\bar{x} > 412$?
- c. What is the probability that $393 \leq \bar{x} \leq 407$?
- d. What is the probability that $\bar{x} \leq 389$?

Q4 (6.10). The lifetimes of lightbulbs produced by a particular manufacturer have a mean of 1200 hours and a standard deviation of 400 hours. The population distribution is normal. Suppose that you purchase nine bulbs, which can be regarded as a random sample from the manufacturer's output.

- a. What is the mean of the sample mean lifetime?
- b. What is the variance of the sample mean?
- c. What is the standard error of the sample mean?
- d. What is the probability that, on average, those nine lightbulbs have lives of fewer than 1050 hours?

Q5 (6.14). A random sample of 16 junior managers in the offices of corporations in a large city center was taken to estimate average daily commuting time for all such managers. Suppose that the population times have a normal distribution with a mean of 87 minutes and a standard deviation of 22 minutes.

- a. What is the standard error of the sample mean commuting time?
- b. What is the probability that the sample mean is fewer than 100 minutes?
- c. What is the probability that the sample mean is more than 80 minutes?
- d. What is the probability that the sample mean is outside the range 85 and 95 minutes?

Q6 (6.17). The times spent studying by students in the week before final exams follows a normal distribution with standard deviation 8 hours. A random sample of four students was taken in order to estimate the mean study time for the population of all students.

- a. What is the probability that the sample mean exceeds the population mean by more than 2 hours?
- b. What is the probability that the sample mean is more than 3 hours below the population mean?
- c. What is the probability that the sample mean differs from the population mean by more than 4 hours?

Q7 (6.24). An English literature course was taken by 250 students. Each member of a random sample of 50 of these students was asked to estimate the amount of time she spent on the previous week's assignment. Suppose that the population standard deviation is 30 minutes.

- a. What is the probability that the sample mean exceeds the population mean by more than 2.5 minutes?
- b. What is the probability that the sample mean is more than 5 minutes below the population mean?
- c. What is the probability that the sample mean differs from the population mean by more than 10 minutes?

Q8 (6.48). A random sample of size $n = 25$ is obtained from a normally distributed population with a population mean of 198 and a variance of 100.

- a. What is the probability that the sample mean is greater than 200?
- b. What is the value of the sample variance such that 5% of the sample variances would be less than this value?
- c. What is the value of the sample variance such that 5% of the sample variances would be greater than this value?

Q9 (6.52). It is believed that first-year salaries for newly qualified accountants follow a normal distribution with a standard deviation of \$2,500. A random sample of 16 observations was taken.

- a. Find the probability that the sample standard deviation is more than \$ 3,000.
- b. Find the probability that the sample standard deviation is less than \$ 1,500.

Q10 (6.54). In a large city it was found that summer electricity bills for single-family homes followed a normal distribution with a standard deviation of \$100. A random sample of 25 bills was taken.

- a. Find the probability that the sample standard deviation is less than \$75.
- b. Find the probability that the sample standard deviation is more than \$ 150.

Q11 (6.55). The number of hours spent watching television by students in the week before final exams has a normal distribution with a standard deviation of 4.5 hours. A random sample of 30 students was taken.

- a. Is the probability more than 0.95 that the sample standard deviation exceeds 3.5 hours?
- b. Is the probability more than 0.95 that the sample standard deviation is less than 6 hours?

Q12 (6.59). Each member of a random sample of 15 business economists was asked to predict the rate of inflation for the coming year. Assume that the predictions for the whole population of business economists follow a normal distribution with standard deviation 1.8%.

- a. The probability is 0.01 that the sample standard deviation is bigger than what number?
- b. The probability is 0.025 that the sample standard deviation is smaller than what number?
- c. Find any pair of numbers such that the probability that the sample standard deviation lies between these numbers is 0.90.

Q13 (6.60). A precision instrument is checked by making 12 readings on the same quantity. The population distribution of readings is normal.

- a. The probability is 0.95 that the sample variance is more than what percentage of the population variance?
- b. The probability is 0.90 that the sample variance is more than what percentage of the population variance?
- c. Determine any pair of appropriate numbers, a and b, to complete the following sentence: The probability is 0.95 that the sample variance is between a % and b% of the population variance.

Q14 (6.70). Refer to the chapter appendix in order to derive the mean of the sampling distribution of the sample variances for a sample of n observations from a population of N members when the population variance is σ^2 . By assuming the sample size n is the large proportion of the population size N , show that $E(s^2) = N\sigma^2/(N - 1)$.