ECO239 HW Questions for "Discrete Random Variables and probability Distributions" Part 2

Q1. (This is the question we covered during the class and I promised you to post the answer.) A market researcher wants to determine whether a new model of a personal computer that had been advertised on a late-night talk show had achieved more brand-name recognition among people who watched the show regularly than among people who did not. After conducting a survey, it was found that 15% of all people both watched the show regularly and could correctly identify the product. Also, 16% of all people regularly watched the show and 45% of all people could correctly identify the product. Define a pair of random variables as follows.

| | |
|---|---|
| X = 1 if regularly watch the show | X = 0 otherwise |
| Y = 1 if product correctly identified | Y = 0 otherwise |

a. Find the joint probability function of X and Y
b. Find the conditional probability function of Y, given X = 1.
c. Find and interpret the covariance between X and Y.

Binomial Distribution

Q2. Suppose that the probability is 0.5 that the value of the U.S. dollar will rise against the Japanese yen over any given week and that the outcome in one week is independent of that in any other week. What is the probability that the value of the U.S. dollar will rise against the Japanese yen in a majority of weeks over a period of 7 weeks? [Note: Use Binomial table to answer this question.]

Q3. A small commuter airline flies planes that can seat up to eight passengers. The airline has determined that the probability that a ticketed passenger will not show up for a flight is 0.2. For each flight the airline sells tickets to the first 10 people placing orders. The probability distribution for the number of tickets sold per flight is shown in the accompanying table. For what proportion of the airline's flights does the number of ticketed passengers showing up exceed the number of available seats? (Assume independence between the number of tickets sold and the probability that a ticketed passenger will show up.) [Note: Use Binomial table to answer this question.]

| Number of tickets | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Probability | 0.25 | 0.35 | 0.25 | 0.10 | 0.05 |

Hypergeometric Distribution

Q4. Compute the probability of 9 successes in a random sample of size n=20 obtained from a population of size N=80 that contains 42 successes.

Q5. A committee of eight members is to be formed from a group of eight men and eight women. If the choice of committee members is made randomly, what is the probability that precisely half of these members will be women?

Q6. A bank executive is presented with loan applications from 10 people. The profiles of the applicants are similar, except that 5 are minorities and 5 are not minorities. In the end the executive approves 6 of the applications. If these 6 approvals are chosen at random from the 10 applications, what is the probability that less than half the approvals will be of applications involving minorities?

Poisson distribution

Q7. A professor receives, on average, 4.2 telephone calls from students the day before a final examination. If the distribution of calls is Poisson, what is the probability of receiving at least three of these calls on such as day?

Q8. The Internal Revenue Service reported that 5.5% of all taxpayers filling out the 1040 short from make mistakes. If 100 of these forms are chosen at random, what is the probability that fewer than 3 of them contain errors? Use the Poisson approximation to the binomial distribution.

Joint Distribution, Covariance, Correlation etc.

Q9. A college bookseller makes calls at the offices of professors and forms the impression that professors are more likely to be away from their offices on Friday than any other working day. A review of the records of calls, one-fifth of which are on Fridays, indicates that for 16%  OF Friday calls, the professor if away from the office, while this occurs for only 12% of calls on every other working day. Define the random variables as follows.

| | |
|---|---|
| X = 1 if call is made on a Friday | X = 0 otherwise |
| Y = 1 if professor is away from the office | Y = 0 otherwise |

a. Find the joint probability function of X and Y.
b. Find the conditional probability function of Y, given X = 0.
c. Find the marginal probability functions of X and Y.
d. Find and interpret the covariance between X and Y.

Q10. A company has 5 representatives covering large territories and 10 representatives covering smaller territories. The probability distributions for the numbers of orders received by each of these types of representatives in a day are shown in the accompanying table. Assuming that the number of orders received by any representative is independent of the number received by any other, find the mean and standard deviation of the total number of orders received by the company in a day.

| # of orders (Large territory) | Probability | # of orders (Smaller Territory) | Probability |
|---|---|---|---|
| 0 | 0.08 | 0 | 0.18 |
| 1 | 0.16 | 1 | 0.26 |
| 2 | 0.28 | 2 | 0.36 |
| 3 | 0.32 | 3 | 0.13 |
| 4 | 0.10 | 4 | 0.07 |
| 5 | 0.06 | | |

Chapter Exercises

Q11. It is estimated that 55% of the freshmen entering a particular college will graduate from that college four years.

a. For a random sample of five entering freshmen, what is the probability that exactly three will graduate in four years?
b. For a random sample of five entering freshmen, what is the probability that a majority will graduate in four years?
c. Eighty entering freshmen are chosen at random. Find the mean and standard deviation of the proportion of these 80 that will graduate in four years.

Q12. Consider a country that imports steel and exports automobiles. The value per unit of cars exported is measured in units of thousands of dollars per car by the random variable X. The value per unit of steel imported is measured in units of thousands of dollars per ton of steel by the random variable Y. Suppose that the country annually exports 10 cars and 5 tons of steel. Compute the mean and variance of the trade balance where the trade balance is the total dollars received for all cars exported minus the total dollars spent for all steel imported. The joint probability distribution for the prices of cars and steel is shown in the table below.

| | Price of Automobiles (X) | | |
|---|---|---|---|
| Price of Steel (Y) | \$ 3 | \$ 4 | \$ 5 |
| \$ 4 | 0.10 | 0.15 | 0.05 |
| \$ 6 | 0.10 | 0.20 | 0.10 |
| \$ 8 | 0.05 | 0.15 | 0.10 |