Discrete Probability Distribution

Week 9

Random variables

A *random variable* is a numeric quantity whose value depends on the outcome of a random event

- We use a capital letter, like X, to denote a random variable
- The values of a random variable are denoted with a lowercase letter, in this case x
- For example, *P(X = x)*

- There are two types of random variables:
- *Discrete random variables* often take only integer values
  - Example: Number of credit hours, Difference in number of credit hours this term vs last
- *Continuous random variables* take real (decimal) values
  - Example: Cost of books this term, Difference in cost of books this term vs last

Discrete Random Variable

- can only take on a countable number of values.

- e.g. Roll a dice twice
- X: # of time 4 comes up

$\Rightarrow$X = 0, 1 or 2.

- e.g. Toss a coin 5 times
- X: # of heads
- =>X = 0,1,2,3,4,or 5

Discrete Probability Distribution

- Experiments: Toss 2 coins, X: # of heads
- (X = 0, 1, or 2).
- 4 possible outcomes

1st toss	2nd toss
H	H
H	T
T	H
T	T

=>

X	Probability	
0	¼ (T,T)	= 0.25
1	2/4 (T,H) (H,T)	= 0.5
2	¼ (H,H)	= 0.25

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Probability Distribution Function

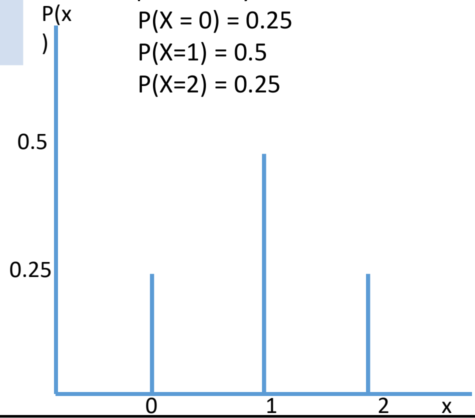
$P(x) = P(X = x)$

The probability of "X takes the value x".

$P(X = 0) = 0.25$

$P(X=1) = 0.5$

$P(X=2) = 0.25$



Cumulative Probability Function

- $F(X_0) = P(X \leq x_0)$
- = The probability that "X is less than or equal to x0".

$$F(X_0) = \sum_{X \leq x0} P(X)$$

- e.g.

X	P(X)	F(X0)	
0	0.25	0.25	$P(X \leq 0) = P(x=0) = F(0)$
1	0.5	0.75	$P(X \leq 1) = P(X=0) + P(X=1) = F(1)$
2	0.25	1.0	$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = F(2)$

Expected Value (= Mean)

- $E(X) = \mu = \sum xP(x)$
- e.g. Calculate the expected value

X	P(X)
0	0.25
1	0.5
2	0.25

- $E(x) = 0*0.25 + 1*0.5+$

Expectation

- We are often interested in the average outcome of a random variable.
- We call this the *expected value* (mean), and it is a weighted average of the possible outcomes

$$\mu = E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

Expected value of a discrete random variable

In a game of cards you win \$1 if you draw a heart, \$5 if you draw an ace (including the ace of hearts), \$10 if you draw the king of spades and nothing for any other card you draw. Write the probability model for your winnings, and calculate your expected winning.

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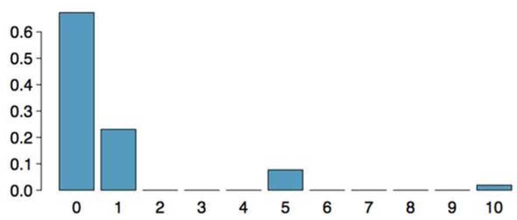
Event	X	$P(X)$	$X\,P(X)$
Heart (not ace)	1	$\frac{12}{52}$	$\frac{12}{52}$
Ace	5	$\frac{4}{52}$	$\frac{20}{52}$
King of spades	10	$\frac{1}{52}$	$\frac{10}{52}$
All else	0	$\frac{35}{52}$	0
Total			$E(X) = \frac{42}{52} \approx 0.81$

What is the meaning of 0.81???

Expected value of a discrete random variable (cont.)

Below is a visual representation of the probability distribution of winnings from this game:



Variance and Standard Deviation

$$\sigma^2 = E(x-\mu)^2 = \sum_{i=1}^{k}(x_i - E(X))^2 P(X = x_i)$$

$$= E(x^2) - \mu_x^2 = \sum_x x^2 P(x) - \mu_x^2$$

$$\sigma = SD(X) = \sqrt{Var(X)}$$

$$\sigma^2 = E(x-\mu)^2 = \sum_x x^2 P(x) - \mu_x^2$$

Practice

$$= \sum_{i=1}^{k}(x_i - E(X))^2 P(X = x_i)$$

- Calculate Variance

X	P(X)
0	0.25
1	0.5
2	0.25

μ = 1
σ2 = (0-1)^2 * 0.25
+ (1-1)^2 * 0.5
+ (2-1)^2*0.25
= 0.5 (Variance)
σ = 0.707 (St.dev)

Practice

- Calculate E(X) and Variance

X	1	2	3	4	5
P(X)	0.07	0.19	0.28	0.30	0.16

E(X) = 1*0.07 + 2*0.19+3*0.28+4*0.30+5*0.16
=3.29
Var(X) = (1-3.29)^2 * 0.07 + (2-3.29)^2 * 0.19 +
(3 – 3.29)^2 * 0.28 + (4- 3.29)^2 * 0.30
+ (5-3.29)^2 * 0.16
= 1.3259

Variability of a discrete random variable

For the previous card game example, how much would you expect the winnings to vary from game to game?

X	$P(X)$	$X\,P(X)$
1	$\frac{12}{52}$	$1 \times \frac{12}{52} = \frac{12}{52}$
5	$\frac{4}{52}$	$5 \times \frac{4}{52} = \frac{20}{52}$
10	$\frac{1}{52}$	$10 \times \frac{1}{52} = \frac{10}{52}$
0	$\frac{35}{52}$	$0 \times \frac{35}{52} = 0$
		$E(X) = 0.81$

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		$E(X) = 0.81$

Variability of a discrete random variable

For the previous card game example, how much would you expect the winnings to vary from game to game?

X	$P(X)$	$X\,P(X)$	$(X - E(X))^2$	$P(X)\,(X - E(X))^2$
1	$\frac{12}{52}$	$1 \times \frac{12}{52} = \frac{12}{52}$	$(1 - 0.81)^2 = 0.0361$	$\frac{12}{52} \times 0.0361 = 0.0083$
5	$\frac{4}{52}$	$5 \times \frac{4}{52} = \frac{20}{52}$	$(5 - 0.81)^2 = 17.5561$	$\frac{4}{52} \times 17.5561 = 1.3505$
10	$\frac{1}{52}$	$10 \times \frac{1}{52} = \frac{10}{52}$	$(10 - 0.81)^2 = 84.4561$	$\frac{1}{52} \times 84.0889 = 1.6242$
0	$\frac{35}{52}$	$0 \times \frac{35}{52} = 0$	$(0 - 0.81)^2 = 0.6561$	$\frac{35}{52} \times 0.6561 = 0.4416$
		$E(X) = 0.81$		$V(X) = 3.4246$ $SD(X) = \sqrt{3.4246} = 1.85$

Linear combinations

- A *linear combination* of random variables X and Y is given by

$$aX + bY$$

  - where a and b are some fixed numbers.

Linear combinations

- A *linear combination* of random variables X and Y is given by $aX + bY$
  - where a and b are some fixed numbers.
- The average value of a linear combination of random variables is given by

$$E(aX + bY) = a \times E(X) + b \times E(Y)$$

Calculating the expectation of a linear combination

On average you take 10 minutes for each statistics homework problem and 15 minutes for each economics homework problem. This week you have 5 statistics and 4 economics homework problems assigned. What is the total time you expect to spend on statistics and economics homework for the week?

Calculating the expectation of a linear combination

On average you take 10 minutes for each statistics homework problem and 15 minutes for each chemistry homework problem. This week you have 5 statistics and 4 chemistry homework problems assigned. What is the total time you expect to spend on statistics and physics homework for the week?

$$
\begin{aligned}
E(5S + 4C) &= 5 \times E(S) + 4 \times E(C) \\
&= 5 \times 10 + 4 \times 15 \\
&= 50 + 60 \\
&= 110\ min
\end{aligned}
$$

Linear Combination

The variability of a linear combination of two independent random variables is calculated as:

$$V(aX + bY) = a^2 \times V(X) + b^2 \times V(Y)$$

Linear Combination

- The variability of a linear combination of two independent random variables is calculated as:

$$V(aX + bY) = a^2 \times V(X) + b^2 \times V(Y)$$

- The standard deviation of the linear combination is the square root of the variance.

Some rules of Exp and Var.

- $E(a)= a$ where a is a constant.
- $Var(a) = 0$

- $E(bX)=bE(X)=b\mu_x$
- $Var(bX)=$ $b^2\sigma_x^2$
- $\mu_x=E(x)=\sum xP(x)$
- $Var(x)=E(x-\mu)^2 = \sum(x-\mu x)^2*P(x)$
- $=\sum x^2 P(x) - \mu_x^2$
-

Y=g(x)=a+bX where a, b: constant, X is Random Variable.

- E(Y)= a + b*μx

$$\sigma_Y^2 = Var(a + bx) = b^2\sigma_x^2$$

Linear combinations

The standard deviation of the time you take for each statistics homework problem is 1.5 minutes, and it is 2 minutes for each economics problem. What is the standard deviation of the time you expect to spend on statistics and economics homework for the week if you have 5 statistics and 4 economics homework problems assigned?

Linear combinations

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$$\begin{aligned} V(5S + 4C) &= 5^2 \times V(S) + 4^2 \times V(C) \\ &= 25 \times 1.5^2 + 16 \times 2^2 \\ &= 56.25 + 64 \\ &= 120.25 \end{aligned}$$

SD(5S+4C)=sqrt(120.25)=10.97

Practice

- C = 25000 + 900 X
- (Cost function. X is # of days for completing a project)

X	P(X)
10	0.1
11	0.3
12	0.3
13	0.2
14	0.1

Find E(X) and Var(X)

Find E(C) and Var(C)

- E(X)=11.9 days
- Var(X)=1.29

- E(C)= E(25000 + 900X) = 25000+900(11.9) =35,710.

- Var(C)=Var(25000 + 900X)=900^2 Var(X)
- = (900)^2 * (1.29) = 1044900.

Discrete Probability Distribution

- Bernoulli Distribution
- Binomial Distribution
- Hypergeometric Distribution
- Poisson Distribution

1. Bernoulli Distribution

- Consider only two outcomes
- “success” or “failure”

- P : probability of success
- 1-P : probability of failure
- X = 1 if success, x = 0 if failure.

- Bernoulli Probability Function
- P(0) = 1-P
- P(1) = P

- Consider E(X) and Var(X) if X ~ Bernoulli
- E(X) = $\sum xP(x) = 0*(1-P) + 1*(P) = P$
- $Var(X) = E[(x-\mu)^2] = \sum (x-\mu)^2 * P(x) = P(1-P)$.

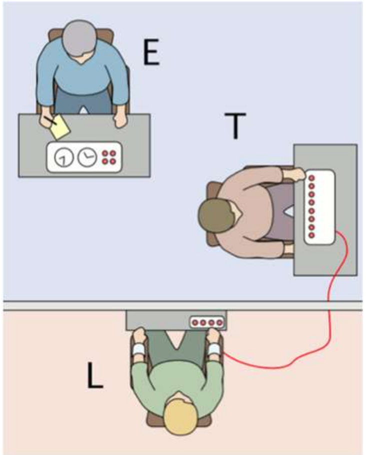
Practice

- P(X: making a sale) = 0.4
- 1-P(X) = 0.6.
- Find E(X) and Var(X).
- E(X)=1*0.4 + 0*0.6 = 0.4 = P
- Var(X) = P(1-P) = 0.4*0.6=0.24.

Milgram experiment

Stanley Milgram, a Yale University psychologist, conducted a series of experiments on obedience to authority starting in 1963.

- Experimenter (E) orders the teacher (T), the subject of the experiment, to give severe electric shocks to a learner (L) each time the learner answers a question incorrectly.
- The learner is actually an actor, and the electric shocks are not real, but a pre-recorded sound is played each time the teacher administers an electric shock.



http://en.wikipedia.org/wiki/File:Milgram_Experiment_v2.png

Milgram experiment (cont.)

- These experiments measured the willingness of study participants to obey an authority figure who instructed them to perform acts that conflicted with their personal conscience.
- Milgram found that about 65% of people would obey authority and give such shocks.
- Over the years, additional research suggested this number is approximately consistent across communities and time.

Bernoulli random variables

- Each person in Milgram's experiment can be thought of as a *trial*.
- A person is labeled a *success* if she refuses to administer a severe shock, and *failure* if she administers such shock.
- Since only 35% of people refused to administer a shock, *probability of success* is p = 0.35.
- When an individual trial has only two possible outcomes, it is called a *Bernoulli random variable*.

- E(X) from this experiment = P = 0.35
- Var(X) = P(1-P) = 0.35*0.65 = 0.2275

2. Binomial Probability Distribution

<= n repeated Bernoulli experiments

- A fixed number of observations, n
- (e.g. 15 tosses of a coin)

- Two mutually exclusive and collectively exhaustive categories- “success (P)” and “failure (1-P)”.
- (e.g. Head or Tail, Defective, not Defective..)

- Constant probability for each observation.
- (e.g. probability of getting a tail is the same each time we toss a coin)

- Observations are independent.
- : The outcome of one observation does not affect the outcome of the other.

Binomial Probability Distribution

$$P(X) = \frac{n!}{x!\,(n-x)!} P^{x}(1-P)^{n-x}$$

- P(X): probability of x successes in n trials, with P = probability of success.

- X: # of success in sample. (x= 0, 1, 2, …n)
- n: sample size
- P: probability of success.

Practice $P(X) = \frac{n!}{x!\,(n-x)!} P^{x}(1-P)^{n-x}$

- Flip a coin 4 times. X = # of heads.

- Q: What is the probability of 3 heads out of 4 flips?

- n = 4
- P=0.5
- X=3

$$(X = 3) = \frac{4!}{3!\,(4-3)!} 0.5^{3}(1-0.5)^{(4-1)}$$

= 0.25

Why use Combination?

$$\frac{n!}{x!\,(n-x)!}$$

- There are four possible scenarios for 3 Hs out of 4 flips. 4!/(3!1!) = 4.

	1st flip	2nd flip	3rd flip	4th flip
1st scenario	H	H	H	T
2nd scenario	H	H	T	H
3rd scenario	H	T	H	H
4th scenario	T	H	H	H

Binomial distribution :Milgram's Experiment re-visited.

Suppose we randomly select four individuals to participate in this experiment. What is the probability that exactly 1 of them will refuse to administer the shock?

$$P(X) = \frac{n!}{x!\,(n-x)!} P^{x}(1-P)^{n-x}$$

P(X=1| n=4, p=0.35)
=[4!/(1!3!)]*0.35^1 *0.65^3
=4*0.35*0.65^3
=0.388

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The probability of exactly one 1 of 4 people refusing to administer the shock is the sum of all of these probabilities.

$$0.0961 + 0.0961 + 0.0961 + 0.0961 = 4 \times 0.0961 = 0.3844$$

Computing the # of scenarios

Earlier we wrote out all possible scenarios that fit the condition of exactly one person refusing to administer the shock. If *n* was larger and/or *k* was different than 1, for example, *n* = 9 and *k* = 2:

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*RR*SSSSSSS
S*RR*SSSSSS
SS*RR*SSSSS
. . .
SS*R*SS*R*SSS
. . .
SSSSSSS*RR*

writing out all possible scenarios would be incredibly tedious and prone to errors.

Computing the # of scenarios

Choose function

The *choose function* is useful for calculating the number of ways to choose *k* successes in *n* trials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

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$$k = 1, n = 4: \binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4 \times 3 \times 2 \times 1}{1 \times (3 \times 2 \times 1)} = 4$$

Computing the # of scenarios

Choose function

The *choose function* is useful for calculating the number of ways to choose *k* successes in *n* trials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$k = 1, n = 4: \binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4 \times 3 \times 2 \times 1}{1 \times (3 \times 2 \times 1)} = 4$$

$$k = 2, n = 9: \binom{9}{2} = \frac{9!}{2!(9-1)!} = \frac{9 \times 8 \times 7!}{2 \times 1 \times 7!} = \frac{72}{2} = 36$$

Note: You can also use R for these calculations:

```
> choose(9,2)
[1] 36
```

Binomial Distribution
Mean and Variance

- $E(X)=np$
- $Var(X)=E(X-\mu)^2 = np(1-p)$

Expected value

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- Or more formally, $\mu = np$ = 100 x 0.262 = 26.2.

Expected value

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- Easy enough, 100 x 0.262 = 26.2.
- Or more formally, $\mu = np$ = 100 x 0.262 = 26.2.
- But this doesn't mean in every random sample of 100 people exactly 26.2 will be obese. In fact, that's not even possible. In some samples this value will be less, and in others more. How much would we expect this value to vary?

Expected value and its variability

Mean and standard deviation of binomial distribution

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

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Mean and standard deviation of binomial distribution

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

Going back to the obesity rate:

$$\sigma = \sqrt{np(1-p)} = \sqrt{100 \times 0.262 \times 0.738} \approx 4.4$$

Expected value and its variability

Mean and standard deviation of binomial distribution

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

Going back to the obesity rate:

$$\sigma = \sqrt{np(1-p)} = \sqrt{100 \times 0.262 \times 0.738} \approx 4.4$$

We would expect 26.2 out of 100 randomly sampled Americans to be obese, with a standard deviation of 4.4.

Note: Mean and standard deviation of a binomial might not always be whole numbers, and that is alright, these values represent what we would expect to see on average.

Unusual observations

Using the notion that *observations that are more than 2 standard deviations away from the mean are considered unusual* and the mean and the standard deviation we just computed, we can calculate a range for the plausible number of obese Americans in random samples of 100.

26.2 ± (2 x 4.4) → (17.4, 35.0)

Practice

An August 2012 Gallup poll suggests that 13% of Americans think home schooling provides an excellent education for children. Would a random sample of 1,000 Americans where only 100 share this opinion be considered unusual?

A. Yes B. No

	Excellent	Good	Only fair	Poor	Total excellent/ good
	%	%	%	%	%
Independent private school	31	47	13	2	78
Parochial or church-related schools	21	48	18	5	69
Charter schools	17	43	23	5	60
Home schooling	13	33	30	14	46
Public schools	5	32	42	19	37

Gallup, Aug. 9-12, 2012

http://www.gallup.com/poll/156974/private-schools-top-marks-educating-children.aspx

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$$\mu = np = 1,000 \times 0.13 = 130$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{1,000 \times 0.13 \times 0.87} \approx 10.6$$

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Range of usual observations: $130 \pm 2 \times 10.6 = (108.8, 151.2)$

100 is outside this range, so would be considered unusual.

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Practice

Which of the following is not a condition that needs to be met for the binomial distribution to be applicable?

A. the trials must be independent
B. the number of trials, *n*, must be fixed
C. each trial outcome must be classified as a *success* or a *failure*
D. the number of desired successes, *k*, must be greater than the number of trials
E. the probability of success, *p*, must be the same for each trial

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Practice

A 2012 Gallup survey suggests that 26.2% of Americans are obese. Among a random sample of 10 Americans, what is the probability that exactly 8 are obese?

A. pretty high
B. pretty low

Gallup: http://www.gallup.com/poll/160061/obesity-rate-stable-2012.aspx, January 23, 2013.

Practice

A 2012 Gallup survey suggests that 26.2% of Americans are obese. Among a random sample of 10 Americans, what is the probability that exactly 8 are obese?

(a) $0.262^8 \times 0.738^2$

(b) $\binom{8}{10} \times 0.262^8 \times 0.738^2$

(c) $\binom{10}{8} \times 0.262^8 \times 0.738^2$

(d) $\binom{10}{8} \times 0.262^2 \times 0.738^8$

Practice

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(b) $\binom{8}{10} \times 0.262^8 \times 0.738^2$

(c) $\binom{10}{8} \times 0.262^8 \times 0.738^2 = 45 \times 0.262^8 \times 0.738^2 = 0.0005$

(d) $\binom{10}{8} \times 0.262^2 \times 0.738^8$

Practice

- 40% of students admitted to university A will actually enroll.

a. What is the probability that at most 1 student will enroll if the college offers admission to 5 students?

$$P(X) = \frac{n!}{x!\,(n-x)!} P^x (1-P)^{n-x}$$

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$$P(X) = \frac{n!}{x!\,(n-x)!} P^{x}(1-P)^{n-x}$$

- n=5
- p=0.4
- X: at most 1 ($X \leq 1$)

- $P(X \leq 1|n=5,p=0.4) = P(X=0)+P(X=1)$
- =[5!/(0!5!)]*(0.4^0)*(0.6^5)
- +[5!/(1!4!)]*(0.4^1)*(0.6^4)
- =0.337