Discrete Probability Distribution

Week 9

Discrete Random Variable

- -can only take on a countable number of values.

- e.g. Roll a dice twice
- X: # of time 4 comes up

⇒X = 0, 1 or 2.

- e.g. Toss a coin 5 times
- X: # of heads
- =>X = 0,1,2,3,4,or 5

Discrete Probability Distribution

- Experiments: Toss 2 coins, X: # of heads
- (X = 0, 1, or 2).
- 4 possible outcomes

1st toss	2nd toss
H	H
H	T
T	H
T	T

=>

X	Probability	
0	¼ (T,T)	= 0.25
1	2/4 (T,H) (H,T)	= 0.5
2	¼ (H,H)	= 0.25

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Probability Distribution Function

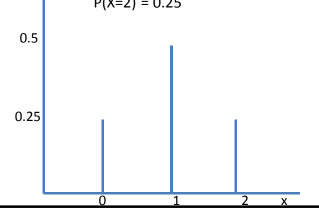
P(x) = P(X = x)

The probability of "X takes the value x".

P(X = 0) = 0.25

P(X=1) = 0.5

P(X=2) = 0.25



Cumulative Probability Function

- $F(X_0) = P(X \leq x_0)$
- = The probability that "X is less than or equal to x0".

$$F(X_0) = \sum_{X \leq x0} P(X)$$

- e.g.

X	P(X)	F(X0)	
0	0.25	0.25	$P(X \leq 0)=P(x=0) = F(0)$
1	0.5	0.75	$P(X \leq 1)=P(X=0)+P(X=1) = F(1)$
2	0.25	1.0	$P(X \leq 2)=P(X=0)+P(X=1)+P(X=2) = F(2)$

Expected Value (= Mean)

- E(X) = μ = ∑xP(x)
- e.g. Calculate the expected value

X	P(X)
0	0.25
1	0.5
2	0.25

- E(x) = 0*0.25 + 1*0.5+2*0.25=1.0

Variance and Standard Deviation

$$\sigma^2 = E(x-\mu)^2$$
$$= E(x^2) - \mu_x^2$$
$$= \sum_x x^2 P(x) - \mu_x^2$$

Derivation will be explained on the board.

$$\sigma^2 = E(x-\mu)^2$$

Practice

$$= \sum_x x^2 P(x) - \mu_x^2$$

- Calculate Variance

X	P(X)
0	0.25
1	0.5
2	0.25

μ = 1
σ2 = (0-1)^2 * 0.25
+ (1-1)^2 * 0.5
+ (2-1)^2*0.25
= 0.5 (Variance)
σ = 0.707 (St.dev)

Practice

- Calculate E(X) and Variance

X	1	2	3	4	5
P(X)	0.07	0.19	0.28	0.30	0.16

E(X) = 1*0.07 + 2*0.19+3*0.28+4*0.30+5*0.16
=3.29
Var(X) = (1-3.29)^2 * 0.07 + (2-3.29)^2 * 0.19 +
(3 – 3.29)^2 * 0.28 + (4- 3.29)^2 * 0.30
+ (5-3.29)^2 * 0.16
= 1.3259

Function of Random Variables

- P(X) : Probability function of a discrete random variable
- g(X) : Some function of x
- E[g(x)] = ∑ g(x)P(x)
- e.g. For Y=g(x) = a + bX, derive the expressions for E(x) and Var(x).

Y=g(x)=a+bX where a, b: constant, X is Random Variable.

- E(Y)= a + b*μx

$$\sigma_Y^2 = Var(a + bx) = b^2\sigma_x^2$$

We derive these on board.

Some rules of Exp and Var.

- E(a)= a   where a is a constant.
- Var(a) = 0
- E(bX)=bE(X)=bμ_x
- Var(bX)=$b^2\sigma_x^2$

- μ_x=E(x)=$\sum$xP(x)
- Var(x)=E(x-μ)^2 = $\sum$(x-μx)^2*P(x)
-                     =$\sum$x^2 P(x) – μ_x^2

Practice

- C = 25000 + 900 X
- (Cost function. X is # of days for completing a project)

X	P(X)
10	0.1
11	0.3
12	0.3
13	0.2
14	0.1

Find E(X) and Var(X)

Find E(C) and Var(C)

- E(X)=11.9 days
- Var(X)=1.29

- E(C)= E(25000 + 900X) = 25000+900(1.29) =35,710.

- Var(C)=Var(25000 + 900X)=900^2 Var(X)
- = (900)^2 * (1.29) = 1044900.