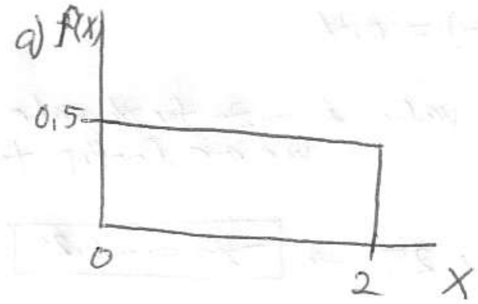
ECO 239 HW6 Answers "Continuous R.Vs and Probability Distribution"

Q1. $X \sim U(0,2)$

a)



$F(X) = \frac{X}{2}$

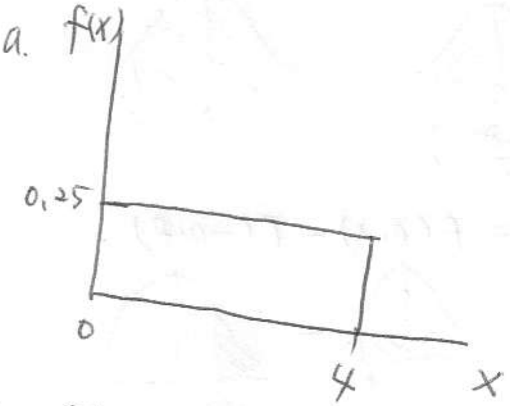
b. $P(1.4 < X < 1.8) = F(1.8) - F(1.4)$
$= \frac{1.8}{2} - \frac{1.4}{2} = \frac{0.4}{2} = \underline{0.2}$

c. $P(1.0 < X < 1.9) = F(1.9) - F(1.0)$
$= \frac{1.9}{2} - \frac{1}{2} = \frac{0.9}{2} = \underline{0.45}$

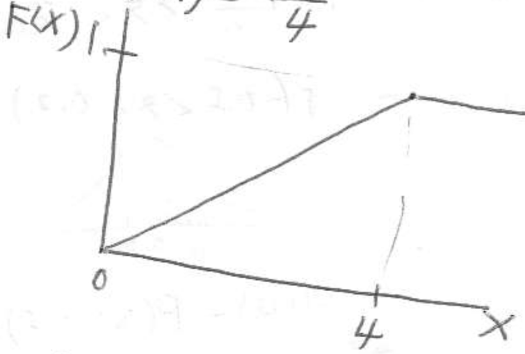
d. $P(X < 1.4) = F(1.4) = \frac{1.4}{2} = \underline{0.7}$

e. $P(X > 1.3) = 1 - P(X < 1.3) = 1 - F(1.3)$
$= 1 - \frac{1.3}{2} = \underline{0.35}.$

Q2. a.

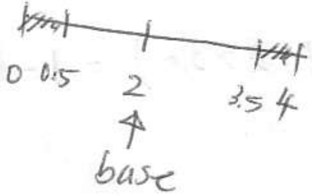


b. $F(X) = \frac{X}{4}$



c. $P(X < 1) = F(1) = \frac{1}{4} = \underline{0.25}.$

d.



$P(0 < X < 0.5) + P(3.5 < X < 4)$
$= 1 - P(0.5 < X < 3.5) = 1 - [F(3.5) - F(0.5)]$
$= 1 - \left(\frac{3.5}{4} - \frac{0.5}{4}\right) = \underline{0.25}.$

Q3. $W = 1000 - 2X$
$\mu_X = 50, \ \sigma_X^2 = 90$

$E_W = 1000 - 100 = 900$
$\sigma_W^2 = 4(90) = 360$

Q4. $W = 6000 + 0.08X$
$\mu_X = 600000$
$\sigma_X = 180000$

$E_W = 6000 + 0.08(600000) = 54000$
$\sigma_W = (0.08)(180000) = 144000$

Q5. a. $P(Z < Z_0) = 0.7$ → Find 0.2 from the Standard Normal Table
* Read inside the table

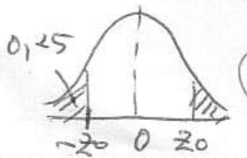
$\boxed{Z_0 = 0.52}$

$Z_0 = 0.52, \ P = 0.6985$
$Z_0 = 0.53, \ P = 0.7019$
} compare & choose the closest to 0.7.

b. $P(Z < -Z_0) = 0.25$ ⇒ $P(Z > Z_0) = 0.25$ ⇒ $1 - P(Z < Z_0) = 0.25$

Negative side

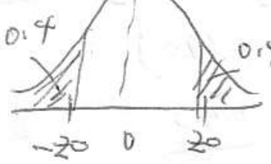
⇒ $P(Z < Z_0) = 0.75.$
$Z_0 = 0.67$ → $\boxed{-Z_0 = -0.67}$



c. $P(Z>z_0)=0.2 \rightarrow 1-P(Z<z_0)=0.2 \rightarrow P(Z<z_0)=0.8$

$\boxed{z_0 = 0.84}$

d. $P(Z>z_0)=0.6 \rightarrow 1-P(Z<z_0)=0.6 \rightarrow P(Z<z_0)=0.4$



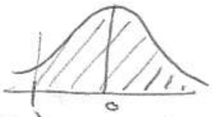
Re-define z_0 as z_0 for positive value & $-z_0$ for the value we are looking for.

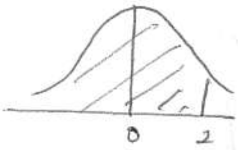
$P(Z>z_0)=0.4$

$=1-P(Z<z_0)=0.4$

$P(Z<z_0)=0.6 \Rightarrow z_0 = 0.25 \rightarrow \boxed{-z_0 = -0.25}$

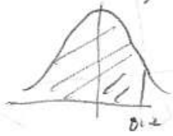
Q6. a. $P(X>60) = P\left(Z>\frac{60-80}{10}\right) = P(Z>-2) = P(Z<2) = \boxed{0.9772}$





b. $P(72<X<82) = P\left(\frac{72-80}{10}<Z<\frac{82-80}{10}\right)$

$= P(-0.8<Z<0.2) = F(0.2)-F(-0.8)$



$= F(0.2) - P(X>0.8) = F(0.2) - [1-F(0.8)]$

$= 0.5793 - 1 + 0.7881 = 0.3674$

c. $P(X<55) = P\left(Z<\frac{55-80}{10}\right) = P(Z<-2.5) = P(Z>2.5) = 1-P(Z<2.5)$

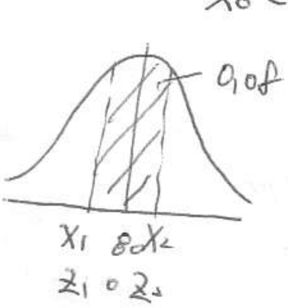
$= 1-F(2.5) = 1-0.9938 = 0.0062.$

d. $P(X>X_0) = 0.1$

$P(Z>z_0)=0.1 \rightarrow P(Z<z_0) = 0.9 \rightarrow z_0 = 1.28$

$\rightarrow X_0 = \mu + \sigma z = 80 + 10(1.28) = 92.8.$

e.



$P(0<Z<z_2) = 0.04 \rightarrow z_2 = 0.10$

$\boxed{-0.1 < Z < 0.1}$

$\Rightarrow 80+10(-0.1) < X < 80+10(0.1)$

$\boxed{79 < X < 81}$

Hw6

③

Q7. $\mu = 380$, $\sigma = 50$

a. $P(X<400) = P\left(Z< \frac{400-380}{50}\right) = P(Z<0.4) = 0.6554$

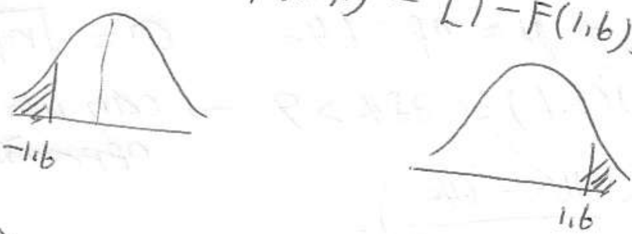
b. $P(X>360) = P\left(Z > \frac{360-380}{50}\right) = P(Z>-0.4) = P(Z<0.4) = 0.6554$

c.

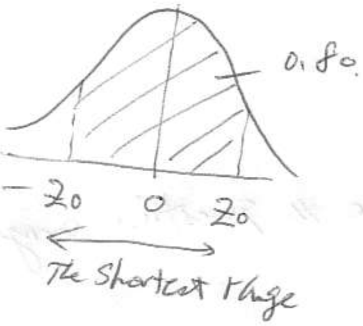
d. $P(300<X<400) = P\left(\frac{300-380}{50} < Z < 0.4\right) = P(-1.6<Z<0.4)$

$= F(0.4) - F(-1.6) = F(0.4) - [1-F(1.6)] = 0.6554 - 1 + 0.9452$

$= 0.6006$



e.



$P(0<Z<z_0) = 0.4$

$z_0 = 1.28$

$\rightarrow$ The range: ± 1.28

$\rightarrow$ $[380 + 50(-1.28)$, $380 + 50(1.28)]$

$\rightarrow$ $\boxed{[\ 316 \quad 444\ ]}$

Q8. $\mu = 35000$, $\sigma = 4000$

a. $P(X>38000) = P\left(Z > \frac{38000-35000}{4000}\right) = P(Z>0.75) = 1 - P(Z<0.75)$

$= 1 - 0.7734 = 0.2266$

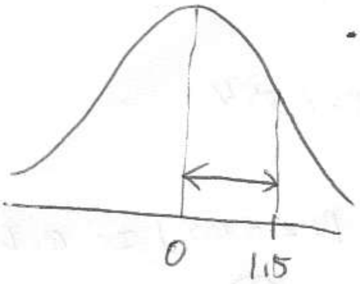
b. $P(X<32000) = P(Z<-0.75) = P(Z>0.75) = 0.2266$

c. $P(32000<X<38000) = P(-0.75<Z<0.75) = 1 - (2 * 0.2266) = 0.5468$

d.

a + c = 1

Q9.

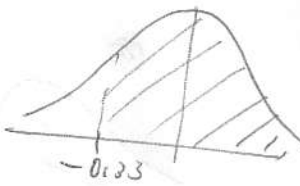


- $z_0 = 1.5$.

$P(Z>1.5) = 1-P(Z<1.5) = 1-0.9332 = 0.0668$.

Q10. A: $P(X>10) = P\left(Z > \frac{10-10.4}{1.2}\right) = P(Z>-0.33)$

$= P(Z<0.33) = F(0.33) = \underline{0.6293}$



B: $P(X>10) = P\left(Z>\frac{10-11}{4}\right) = P(Z>-0.25)$

$= P(Z<0.25) = F(0.25) = 0.5987$

Thefore, Investment A is a better choice.

Q11. $n=1600$, $p=0.4$ $\Rightarrow \mu = np \doteq 640$ $\quad \sigma = \sqrt{np(1-p)} = 19.5959$

$np(1-p) = 1600(0.4)(0.6) = 384 > 9$ → can use Normal approximation.

a. $P(X>1650) = P\left(Z>\frac{1650-640}{19.5959}\right) =$

→ There is a problem in this question

Change $n=1600$ to $n=4000$, then re-solve the question, sorry!

$n = 4000$, $p=0.4$

$\mu = np = 1600$

$\sigma = \sqrt{np(1-p)} = \sqrt{4000(0.4)(0.6)} = 30.98$.

a. $P(X>1650) = P\left(Z>\frac{1650-1600}{30.98}\right) = P(Z>1.61\ldots)$.

$= 1-P(Z<1.61\ldots) = 1-0.9463 = 0.0537$

b. $P(X<1530) = P\left(Z<\frac{1530-1600}{30.98}\right) = P(Z<-2.26) = P(Z>2.26)$

$= 1-P(Z<2.26) = 1-F(2.26)$

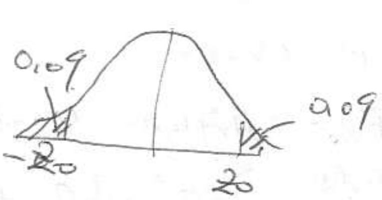
$= 1-0.9881 = 0.0119$

c. $P(1550<X<1650) = P(-1.61<Z<1.61) = F(1.61)-F(-1.61)$

$= F(1.61)-[1-F(1.61)] = 2F(1.61)-1 = 2(0.9463)-1$

$= 0.8926$.

d. $P(Z < -z_0) = 0.09$



$\rightarrow P(Z > z_0) = 0.09$

$\rightarrow 1 - P(Z < z_0) = 0.09 \rightarrow P(Z < z_0) = 0.91 \rightarrow z_0 = 1.34$

$-z_0 = -1.34 \Rightarrow X_0 = \mu + \sigma(-1.34) = 1600 + 30.98(-1.34)$

$= 1558.4868$

e. $P(Z > z_0) = 0.2$

$P(Z < z_0) = 0.8$

$z_0 = 0.84$

$X_0 = 1600 + 30.98(0.84) = 1626.0232$

Q12. $n = 900,\ p = 0.2$

$np(1-p) = 900(0.2)(0.8) = 144 > 9$

$\mu = 900 * 0.2 = 180$

$\sigma = \sqrt{144} = 12$

a. $P(X > 200) = P(Z > \frac{200-180}{12}) = P(Z > 1.67) = 1 - P(Z < 1.67)$

$= 1 - F(1.67) = 1 - 0.9525 = 0.0475$

b. $P(X < 175) = P(Z < \frac{175-180}{12}) = P(Z < -0.42) = P(Z > 0.42)$

$= 1 - P(Z < 0.42) = 1 - 0.6628 = 0.3372$

Q13. $P(Z > \frac{38000-35000}{4000}) = P(Z > 0.75) = 1 - P(Z < 0.75) = 1 - 0.7734 = 0.2266$

$E(X) = 100 * 0.2266 = 22.66$

$\sigma = \sqrt{100(0.2266)(0.7734)} = 4.1863$

$P(Z > \frac{25-22.66}{4.1863}) = P(Z > 0.56) = 1 - P(Z < 0.56) = 1 - F(0.56)$

$= 1 - 0.7123 = \underline{0.2877}$

Q14. $\lambda = \frac{1}{5} = 0.2$

$P(T > 7) = 1 - P(T < 7) = 1 - F(7) = 1 - (1 - e^{-0.2*7}) = e^{-1.4} = 0.2466$

Q15. $\lambda = \frac{1}{15}$

$P(T > 18) = 1 - P(T < 18) = 1 - [1 - e^{-\frac{18}{15}}] = e^{-18/15} = 0.30119$

⑥

Q16. $\mu_X = 100$, $\sigma_X^2 = 100$ $\quad W = 5X + 4Y$

$\mu_Y = 200$, $\sigma_Y^2 = 400$ $\quad E_W = 5\mu_X + 4\mu_Y = 500 + 800 = \underline{\underline{1300}}$

$\rho = -0.5.$ $\quad Var(W) = 25\sigma_X^2 + 16\sigma_Y^2 + 2(5)(4)(-0.5)(10)(20)$

$= 25(100) + 16(400) - 4000 =$

$= \underline{\underline{4900}}$

Q17. $\mu_X = 500$, $\sigma_X^2 = 100$ $\quad W = 5X - 4Y$

$\mu_Y = 200$, $\sigma_Y^2 = 400$ $\quad E_W = 5(500) - 4(200) = 1700$

$\rho = 0.5.$

$Var(W) = 25(100) + 16(400) + 2(5)(-4)(0.5)(10)(20)$

$= \underline{\underline{4900}}$

Q18. $\begin{cases} \mu_{PX} = 25 \\ \sigma_{PX}^2 = 121 \end{cases}$ $\begin{cases} \mu_{PY} = 40 \\ \sigma_{PY}^2 = 225 \end{cases}$ $\rho = 0.5.$

a. $W = 50PX + 40PY$

$E_W = 50(25) + 40(40) = 2850$

$\sigma_W^2 = (50)^2(121) + (40)^2(225) + 2(50)(40)(0.5)(11)(15) = 992,500.$

b. $E_W = 2850$

$\sigma_W^2 = (50)^2(121) + (40)^2(225) + 2(50)(40)\underline{(-0.5)}(11)(15) = 332,500.$

Q19. export price $= P_X$ $\quad \mu_{PX} = 100$, $\sigma_{PX}^2 = 100$

import price $= P_Y$ $\quad \mu_{PY} = 90$, $\sigma_{PY}^2 = 400$ $\quad \rho = -0.4$

$W = 10P_X - 10P_Y$

a. $E_W = 10(100) - 10(90) = 100$

$\sigma_W^2 = (10)^2(100) + (-10)^2(400) + 2(10)(-10)(-0.4)(10)(20)$

$= 66000$ $\quad (\sigma_W = 256.90465)$

b. $P\left(Z < \dfrac{0 - 100}{256.90465}\right) = P(Z < -0.39) = P(Z > 0.39) = 1 - P(Z < 0.39)$

$= 1 - F(0.39) = 1 - 0.6517 = 0.3483$