

HW Questions “Discrete Random Variables and Probability Distributions” part 1
(for Midterm II)

Question 1

American Travel Air has asked you to study flight delays during the week before Christmas at Midway Airport. The random variables X is the number of flights delayed per hour.

| | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| P(x) | 0.10 | 0.08 | 0.07 | 0.15 | 0.12 | 0.08 | 0.10 | 0.12 | 0.08 | 0.10 |

- What is the cumulative probability distribution?
- What is the probability of five or more delayed flights?
- What is the probability of three through seven (inclusive) delayed flights?

Question 2

An automobile dealer calculates the proportion of new cars sold that have been returned various numbers of times for the correction of defects during the warranty period. The results are shown in the table.

| | | | | | |
|-------------------|------|------|------|------|------|
| Number of returns | 0 | 1 | 2 | 3 | 4 |
| Proportion | 0.28 | 0.36 | 0.23 | 0.09 | 0.04 |

- Draw the probability distribution function.
- Calculate and draw the cumulative probability function.
- Find the mean of the number of returns of an automobile for corrections for defects during the warranty period.
- Find the variance of the number of returns of an automobile for corrections for defects during the warranty period.

Question 3

A corporation produces packages of paper clips. The number of clips per package varies, as indicated in the accompanying table.

| | | | | | | | |
|------------------------|------|------|------|------|------|------|------|
| Number of clips | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
| Proportion of packages | 0.04 | 0.13 | 0.21 | 0.29 | 0.20 | 0.10 | 0.03 |

- Draw the probability function.
- Calculate and draw the cumulative probability function.
- What is the probability that a randomly chosen package will contain between 49 and 51 clips (inclusive)?

- d. Two packages are chosen at random. What is the probability that at least one of them contains at least 50 clips?
- e. The cost (in cents) of producing a package of clips is $16+2X$, where X is the number of clips in the package. The revenue from selling the package, however many clips it contains, is \$1.50. If profit is defined as the difference between revenue and cost, find the mean and standard deviation of profit per package.

Question 4

- a. A very large shipment of parts contains 10% defectives. Two parts are chosen at random from the shipment and checked. Let the random variable X denote the number of defectives found. Find the probability function of this random variable.]
- b. A shipment of 20 parts contains 2 defectives. Two parts are chosen at random from the shipment and checked. Let the random variable Y denote the number of defectives found. Find the probability function of this random variable. Explain why your answer is different from that for part (a).
- c. Find the mean and variance of the random variable X in part (a).
- d. Find the mean and variance of the random variable Y in part (b).

Question 5

A factory manager is considering whether to replace a temperamental machine. A review of past records indicates the following probability distribution for the number of breakdowns of this machine in a week.

| | | | | | |
|----------------------|-----|------|------|------|------|
| Number of breakdowns | 0 | 1 | 2 | 3 | 4 |
| Probability | 0.1 | 0.26 | 0.42 | 0.16 | 0.06 |

- a. Find the mean and standard deviation of the number of weekly breakdowns.
- b. It is estimated that each breakdown costs the company \$1,500 in lost output. Find the mean and standard deviation of the weekly cost to the company from breakdowns of this machine.

Question 6

For a Bernoulli random variable with probability of success $P = 0.5$, compute the mean and variance.

Question 7

For a binomial probability function with $P = 0.3$ and $n = 14$, find the probability that the number of successes is equal to 7 and the probability that the number of successes is less than 6.

Question 8

A politician believes that 25% of all macroeconomists in senior positions will strongly support a proposal he wishes to advance. Suppose that this belief is correct and that five senior macroeconomists are approached at random.

- a. What is the probability that at least one of the five will strongly support the proposal?
- b. What is the probability that a majority of the five will strongly support the proposal?

Question 9

The Cubs are to play a series of five games in St. Louis against the Cardinals. For any one game it is estimated that the probability of a Cubs win is 0.4. The outcomes of the five games are independent of one another.

- a. What is the probability that the Cubs will win all five games?
- b. What is the probability that the Cubs will win a majority of the five games?
- c. If the Cubs win the first game, what is the probability that they will win a majority of the five games?
- d. Before the series begins, what is the expected number of Cubs wins in these five games?
- e. If the Cubs win the first game, what is the expected number of Cubs wins in the five games?

Question 10

A campus finance officer finds that, for all parking tickets issued, 78% are paid. The fine is \$2. In the most recent week 620 parking tickets have been issued.

- a. Find the mean and standard deviation of the number of these tickets for which the fines will be paid.
- b. Find the mean and standard deviation of the amount of money that will be obtained from the payment of these fines.

Question 11

The following two acceptance rules are being considered for determining whether to take delivery of a large shipment of components:

- A random sample of 10 components is checked, and the shipment is accepted only if none of them is defective.
- A random sample of 20 components is checked, and the shipment is accepted only if no more than 1 of them is defective.

Which of these acceptance rules has the smaller probability of accepting a shipment containing 20% defectives?