ch 9 p.2 answers

1) demand $P = 200 - 0.2Q$ eq. point: $200 - 0.2Q = 20 + 0.1Q$

supply $P = 20 + 0.1Q$ $180 = 0.3Q$

$Q = 600$ $P = 20 + 0.1 \cdot 600 = 80$

consumer surplus: $\int_0^{600} [200 - 0.2Q - 80]\, dQ = -\frac{(x-1200)x}{10} + C \Big|_0^{600} = -\frac{(600-1200)}{10} \cdot 600 = 36000$

producer surplus: $\int_0^{600} [80 - 20 - 0.1Q]\, dQ = -\frac{(x-1200) \cdot x}{20}\Big|_0^{600} =$

$= -\frac{(600-1200)}{20} \cdot 600 = 18000$

2) demand $P = 6000/(Q+50)$ $\frac{6000}{Q+50} = Q + 10$ $Q = -110$ $Q = 50$

supply $P = Q + 10$

$p = 60$

consumer S.: $\int_0^{50} \frac{6000}{Q+50} - 60\, dQ = 6000 \int \frac{1}{Q+50}\, dQ - 60 \int 1\, dQ$

$\int \frac{1}{Q+50}\, dQ$ $u = Q + 50$, $du = 1\, dQ$ $\int \frac{1}{u}\, du = \ln u = \ln(Q+50)$ $\int 1\, dQ = Q$

Prod. S: $\int_0^{50} 60 - Q - 10\, dQ =$

$= \int_0^{50} (50 - Q)\, dQ = 50Q - \frac{1}{2}Q^2 =$

$= 50 \cdot 50 - \frac{50^2}{2} - 0 = 1250$

so:

$6000 \int \frac{1}{Q+50}\, dQ - 60 \int 1\, dQ =$

$= 6000(\ln|Q+50|) - 60Q + C \Big|_0^{50}$

$= 6000 \ln 2 - 3000$

≈ 1158.9

5) a) $I = \int xe^{-x}dx \qquad f(x) = x \qquad f'(x) = 1$

$g'(x) = e^{-x} \qquad g(x) = -1e^{-x}$

$I = -e^{-x} \cdot x - \int -e^{-x}dx = -x \cdot e^{-x} + (-(-e^{-x})) = -x \cdot e^{-x} - e^{-x} =$

$= e^{-x}(-x-1) + C$

$= -xe^{-x} - e^{-x} + c$

b) $\int 3xe^{4x}dx \qquad f(x) = 3x \qquad f'(x) = 3$

$g'(x) = e^{4x} \qquad g(x) = \frac{1}{4}e^{4x}$

$3x \cdot \frac{1}{4}e^{4x} - \int \frac{3}{4}e^{4x}dx = 3x \cdot \frac{1}{4}e^{4x} - \frac{3}{4}\int e^{4x} =$

$= \frac{3x}{4}e^{4x} - \frac{3e^{4x}}{16} + c = \frac{3e^{4x}}{16} \cdot (4x-1) + C$

c) $\int (1+x^2)e^{-x}dx \qquad f(x) = (1+x^2) \qquad f'(x) = 2x$

$g'(x) = (e^{-x}) \qquad g(x) = -e^{-x}$

$-e^{-x} \cdot (1+x^2) + \int 2x \cdot (+e^{-x}) = -e^{-x}(1+x^2) - 2(x+1)e^{-x} =$

$= -e^{-x}(3+x^2+2x) + C$

d) $\int x\ln x\, dx \qquad f(x) = \ln x \qquad f'(x) = \frac{1}{x}$

$g'(x) = x \qquad g(x) = \frac{x^2}{2}$

$\frac{x^2}{2} \cdot \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \cdot dx = \frac{x^2}{2}\ln x - \int \frac{x}{2}dx = \frac{x^2\ln x}{2} - \frac{x^2}{4}$

$= \frac{2x^2\ln x - x^2}{4} + C = \frac{x^2(2\ln x - 1)}{4} + C$

4) a) $\int x(2x^2+3)^5\,dx$

$u = 2x^2+3$

$\frac{du}{dx} = 4x$

$du = 4x\,dx$

$\frac{1}{4}\int 4x(u)^5\,dx =$

$= \frac{1}{4}\int u^5\,du$

$= \frac{1}{4}\left(\frac{1}{6}u^6\right) =$

$= \frac{1}{24}(2x^2+3)^6 + C$

b) $\int \underbrace{x^2 e^{x^3+2}\,dx}_{=\frac{1}{3}du}$

$u = e^{x^3+2}$

$\frac{du}{dx} = e^{x^3+2}\cdot 3x^2\,dx \Rightarrow \frac{1}{3}du = x^2 e^{x^3+2}\,dx$

$I = \int \frac{1}{3}du = \frac{1}{3}u + c = \frac{1}{3}e^{x^3+2} + c$

c) $\int \frac{\ln(x+2)}{2x+4}\,dx$

$\frac{1}{2}\int \frac{\ln(x+2)}{x+2}\,dx = \frac{1}{2}\int u\,du =$

$= \frac{1}{2}\left(\frac{u^2}{2}\right) = \frac{u^2}{4} = \frac{(\ln(x+2))^2}{4}$

$u = \ln(x+2)$

$du = \frac{1}{x+2}\,dx$

d) $\int \frac{x^3}{(1+x^2)^3}\,dx =$

$u = (1+x^2) \Rightarrow x^2 = (u-1)$

$du = 2x\,dx$

$\frac{1}{2}du = x\,du$

$I = \int \frac{1}{2}\frac{(u-1)}{u^3}\,du$

$= \frac{1}{2}\int \frac{u-1}{u^3}\,du = \frac{1}{2}\int\left(\frac{1}{u^2} - \frac{1}{u^3}\right)du$

$= \frac{1}{2}\left[-u^{-1} - \frac{1}{(-2)}u^{-2}\right] + C$

$= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + C$

5) a) $\int_0^1 x\sqrt{x^2+1}\,dx$.

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}\int_{x=0}^{x=1} \underbrace{2x\,dx}_{du}\,\underbrace{\sqrt{x^2+1}}_{\sqrt{u}} = \frac{1}{2}\int_{u=1}^{u=2} \sqrt{u}\,du =$

$= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_1^2 =$

$= \frac{1}{2}\left(\frac{2}{3}\cdot 2^{\frac{3}{2}} - \frac{2}{3}\right) \simeq$

$\simeq 0.609476..$

→ you can also solve as

$u = \sqrt{x^2+1}$

$u^2 = x^2+1$

$2u\,du = 2x\,dx$

$u\,du = x\,dx$

$\rightarrow I = \int_{\sqrt{0+1}}^{\sqrt{1+1}} u^2\,du$

$= \int_1^{\sqrt{2}} u^2\,du$

$= \frac{1}{3}u^3\Big|_1^{\sqrt{2}}$

$= \frac{1}{3}(2\sqrt{2}-1)$

$= 0.6094$

b) $\int_1^3 \frac{1}{x^2} e^{\frac{2}{x}}\,dx$

$u = \frac{2}{x}$

$\frac{du}{dx} = -\frac{2}{x^2}$

$dx = -\frac{x^2}{2}\,du$

$-\frac{1}{2}\int e^u\,du = -\frac{e^u}{2}\Big|_1^3 =$

$= -\frac{e^{\frac{2}{x}}}{2} + c\,\Big|_1^3 = \frac{e^2 - e^{\frac{2}{3}}}{2} \simeq 2.72$

6) a) $\int_1^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^3}\,dx = \lim_{b\to\infty}\left[\frac{1}{-2}x^{-2}\Big|_1^b\right] =$

$\lim_{b\to\infty} \frac{1}{-2}b^{-2} + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$

b) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\left(2\sqrt{x}\,\Big|_1^b\right) = \lim_{b\to\infty}\left[2\sqrt{b}-2\right] =$

$= +\infty \Rightarrow$ divergent Because it's $+\infty$

c) $\int_{-\infty}^0 e^x\,dx = \lim_{a\to-\infty}\int_a^0 e^x\,dx = \lim_{a\to-\infty}\left(e^x\Big|_a^0\right) = \lim_{a\to-\infty}\left(e^0 - e^a\right) =$

$= 1 - 0 = 1$

7) 1) $\int_0^1 \frac{\ln x}{x^3} dx$ $\quad f(x) = \ln x \quad f' = \frac{1}{x}$

$g'(x) = \frac{1}{x^3} \quad g(x) = -\frac{1}{2x^2}$

so : $-\frac{\ln x}{2x^2} - \int -\frac{1}{2x^3} dx = -\frac{2\ln x + 1}{4x^2}\Big|_0^1 =$

$= -\frac{2\ln 1 + 1}{4} + \frac{2\ln 0 + 1}{4(0)^2}$ → oops... it's impossible so it means the function is divergent

2) $\int_1^\infty \frac{\ln x}{x^3} dx$:

$\lim_{b \to \infty} -\frac{2\ln x + 1}{4x^2}\Big|_1^b = \underbrace{\frac{-2\ln b + 1}{4b^2}}_{0} + \frac{+1}{4} = \frac{1}{4}$