ECO137 HW Questions for Chapter 7 "Derivative in Use" Part 1.

1. Find the first and the second derivative with respect to x by implicit differentiation when
(a) $x - y + 3xy = 2$ (b) $y^5 = x^6$

2. Suppose that y is a differentiable function of x that satisfies the equation
$2x^2 + 6xy + y^2 = 18$. Find y' and y;; at the point (x,y) = (1,2).

3. Suppose y is defined implicitly as a function of x by the following equations, where g is a given differentiable function of one variable. Find an expression for y'.
(a) $xy = g(x) + y^3$ (b) $g(x+y) = x^2 + y^2$ (c) $(xy+1)^2 = f(x^2y)$

4. Consider the equation $AP^{-\alpha}r^{-\beta} = S$, where A, α, β, S are positive constants. Take natural logarithms of both sides and find dP/dr by implicit differentiation. Determine its sign.

5. Let f be defined by $f(x) = \ln(2 + e^{x-3})\ for\ all\ x.$
(a) Show that f is strictly increasing and find the range of f.
(b) Find an expression for the inverse function g of f. Where is g defined?
(c) Verify that f'(3) = 1/g'(f(3)).

6. Find the linear approximations to the following functions about x = 0.
(a) $f(x) = (1+x)^{-1}$ (b) $f(x) = (1+x)^5$ (c) $f(x) = (1-x)^{1/4}$

7. Find the linear approximation to $F(K) = AK^{\alpha}$ about K = 1.

8. The equation $3xe^{xy^2} - 2y = 3x^2 + y^2$ defines y as a differentiable function of x about the point (x,y) = (1,0).
(a) Find the slope of the graph at this point by implicit differentiation.
(b) What is the linear approximation to y about x = 1?

9. Find the Taylor polynomial of degree 2 about x = 0 for $f(x) = 5(\ln(1+x) - \sqrt{1+x})$.

10. Find the quadratic approximation for y about (x,y) = (0,1) when y is defined implicitly as a function of x by the equation $1 + x^3y + x = y^{1/2}$.

11. Let $g(x) = \sqrt[3]{1+x}$.
(a) Find the Taylor polynomial of g(x) fo order 2 about the origin.
(b) For $x \geq 0$ show that $|R_3(x)| \leq 5x^3/81$.

12. A study of transport economics uses the relation $T = 0.4K^{1.06}$, where K is expenditure on building roads, and T is a measure of traffic volume. Find the elasticity of T w.r.t. K. In this model, if expenditure increases by 1%, by what percentage (approximately) does traffic volume increase?

3. The demand D for apples in the U.S. as a function of income r for the period 1927 to 1941 was estimated as $D = Ar^{1.23}$, where A is constant. Find and interpret the elasticity of D w.r.t. r.