ECO137 Homework Questions for Chapter 8 ' Single-Variable Optimization'

1. Let y denote the weekly average quantity of meat produced in Chicago during 1948 (in millions of pounds) and let x be the total weekly work effort (in thousands of house). Nichols estimated the relation as $y = -2.05 + 1.06x - 0.04x^2$. Determine the value of x that maximizes y by studying the sign variation of y'.

2. Find possible extreme points for $g(x) = x^3 \ln x$, x in $(0, \infty)$.

3. Find possible extreme points for $f(x) = e^{3x} - 6e^{x}, x \in (-\infty, \infty)$.

4. Find the maximum of $y = x^2 e^{-x}$ on [0,4].

5. Find the values of x that maximize/minimize the following functions.

(a) $y = \ln x - 5x$, $x>0$. (b) $y = e^x + e^{-2x}$

6. A firm produces $Q = 2\sqrt{L}$ units of a commodity when L units of labor are employed. If the price obtained per unit is 160 Euros, and the price per unit of labor is 40 Euros, what value of L maximizes profits?

7. Find the maximum and minimum of each function over the indicated interval:

(a) $f(x) = -2x-1$ [0,3], (b) $f(x) = x^3-3x+8$ [-1,2], (c) $f(x) = x^5-5x^3$, $[-1, \sqrt{5}]$

8. For the following functions determine all numbers x* in the specified intervals such that f'(x*) = [f(b) - f(a)]/(b-a):

(a) $f(x) = x^2$ in [1, 2] (b) $f(x) = \sqrt{9 + x^2}$ in [0,4]

9. Given Total Revenue $R(Q) = 10Q - (Q^2/1000)$, Total Cost $C(Q) = 5000 + 2Q$, and Q in [0, 10000], find the value of Q that maximizes profits.

10. The price of a firm obtains for a commodity varies with demand Q according to the formula $P(Q) = 18 - 0.006Q$. Total cost is $C(Q) = 0.004Q^2 + 4Q + 4500$.

(a) find the firm's profit and the value of Q which maximizes profit.

(b) find a formula for the elasticity of P(Q) w.r.t. Q, and find the particular value Q* of Q at which the elasticity is equal to -1.

11. Consider the function f defined for all x by $f(x) = x^3 - 12x$. Find the stationary points of f and classify them by using both the first- and second-derivative tests.

12. Determine the possible local extreme points and values for the following functions:

(a) $f(x) = x^3 - 3x + 8$, (b) $f(x) = x^3 + 3x^2 - 2$.

13. Let f be defined for all x by $f(x) = x^3 + (3/2)x^2 - 6x + 10$.

(a) Find the stationary points of f and determine the intervals where f increase.

(b) Find the inflection point for f.

14. Decide where the following functions are convex and determine possible inflection points:

(a) $f(x) = x/(1+x^2)$, (b) $h(x) = xe^x$, (c) $y = \ln x + 1/x$ (d) $y = (x^2+2x)e^{-x}$.