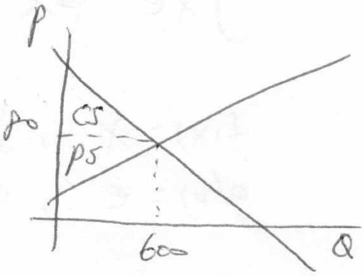
ECO137 (3) Ch 9 Integration part 2. ①



1. $p = 200 - 0.2Q$

$p = 20 + 0.1Q$

$200 - 0.2Q = 20 + 0.1Q$

$0.3Q = 180$

$\begin{cases} Q^* = 180/0.3 = 600 \\ p^* = 200 - \underbrace{0.2(600)}_{120} = 80 \end{cases}$

$$CS = \int_0^{600} 200 - 0.2Q - 80 \, dQ$$

$$= \int_0^{600} 120 - 0.2Q \, dQ$$

$$= 120Q - 0.1Q^2 \Big|_0^{600}$$

$$= 120(600) - 0.1(600^2) = 72000 - 36000 = \underline{36000}$$

$$PS = \int_0^{600} 80 - 20 - 0.1Q \, dQ$$

$$= \int_0^{600} 60 - 0.1Q \, dQ = 60Q - 0.05Q^2 \Big|_0^{600}$$

$$= 60(600) - 0.05(600^2) = 36000 - 18000 = \underline{18000}$$

2. $p = \dfrac{6000}{(Q+50)}, \; P = Q + 10$

$6000 = (Q+50)(Q+10) = Q^2 + 60Q + 500$

$Q^2 + 60Q - 5500 = 0$

$$\frac{-60 \pm \sqrt{3600 + 4(5500)}}{2} = \frac{-60 \pm 160}{2} = \frac{100}{2} = 50$$

$Q^* = 50$

$p^* = 60$

($Q = -110$ is not considered here)

$$CS = \int_0^{50} \frac{6000}{(Q+50)} - 60 \, dQ = 6000 \ln(Q+50) - 60Q \Big|_0^{50}$$

$= 50(50) - \frac{1}{2}(50)^2$

$= 1250$

$$= 6000 \ln(100) - 60(50) - 6000 \ln 50$$

$$= 1158.9$$

$$PS = \int_0^{50} 60 - Q - 10 \, dQ = \int_0^{50} 50 - Q \, dQ = 50Q - \frac{1}{2}Q^2 \Big|_0^{50}$$

3 (a) $\int xe^{-x}dx$

$f(x) = x, \quad f'(x) = 1$
$g'(x) = e^{-x}, \quad g(x) = -e^{-x}$

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + c$$
$$= -(1+x)e^{-x} + c$$

(b) $$\int 3xe^{4x}dx = 3\underbrace{\int xe^{4x}dx}_{I}$$

$f(x) = x \quad f'(x) = 1$
$g'(x) = e^{4x} \quad g(x) = \frac{1}{4}e^{4x}$

$$3I = 3\left[\frac{x}{4}e^{4x} - \frac{1}{4}\int e^{4x}dx\right]$$
$$= \frac{3}{4}\left[xe^{4x} - \frac{1}{4}e^{4x}\right] + c$$

(c) $\int (1+x^2)e^{-x}dx$

$f(x) = (1+x^2) \quad f'(x) = 2x$
$g'(x) = e^{-x} \quad g(x) = -e^{-x}$

$$I = -e^{-x}(1+x^2) + 2\underbrace{\int xe^{-x}dx}_{I_1}$$

$f(x) = x, \; g'(x) = e^{-x}, \; f'(x) = 1, \; g(x) = -e^{-x}$

$$I_1 = -xe^{-x} + \int e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + c$$

$$= -e^{-x}(1+x^2) + 2\left[-xe^{-x} - e^{-x}\right] + c$$
$$= -(x^2+2x+3)e^{-x} + c$$

3 (d) $\int x \ln x \, dx$

$f(x) = \ln x \quad f'(x) = \frac{1}{x}$

$g'(x) = x \quad g(x) = \frac{1}{2}x^2$

$I = \frac{1}{2}x^2 \ln x - \int \frac{1}{2} x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\frac{1}{2}x^2 + c$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$

4. (a) $\int x(2x^2+3)^5 \, dx = \int \frac{1}{4} u^5 \, du = \frac{1}{4}\left[\frac{1}{6}u^6\right] + c$

$u = 2x^2+3$

$du = 4x \, dx \qquad = \frac{1}{24}(2x^2+3)^6 + c$

$\frac{1}{4} du = x \, dx$

(b) $\int x^2 e^{x^3+2} dx = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + c = \frac{1}{3}e^{x^3+2} + c$

$u = x^3+2$

$du = 3x^2 dx$

$\frac{1}{3}du = x^2 dx$

(c) $\int \frac{\ln(x+2)}{2x+4} dx = \frac{1}{2}\int u \, du = \frac{1}{2}\frac{1}{2}u^2 + c = \frac{1}{4}[\ln(x+2)]^2 + c$

$u = \ln(x+2)$

$du = \frac{1}{x+2} dx$

$\frac{1}{2} du = \frac{1}{2x+4} dx$

(d) $\int \frac{x^3}{(1+x^2)^3} dx = \frac{1}{2}\int \frac{(u-1)}{u^3} du = \frac{1}{2}\int u^{-2} - u^{-3} du$

$u = 1 + x^2 \qquad = \frac{1}{2}\left[\frac{1}{-1}u^{-1} - \frac{1}{-2}u^{-2}\right] + c$

$du = 2x \, dx$

$\frac{1}{2}x^2 du = x^3 dx \qquad = -\frac{1}{2}(1+x^2)^{-1} + \frac{1}{4}(1+x^2)^{-2} + c$

$\frac{1}{2}(u-1) du = x^3 dx \qquad = -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + c$

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5. (a) $\int_0^1 x\sqrt{1+x^2}\,dx = \int_{\sqrt{1}=1}^{\sqrt{2}} u^2\,du = \frac{1}{3}u^3\Big|_1^{\sqrt{2}} = \frac{1}{3}(\sqrt{2})^3 - \frac{1}{3}$

$= \frac{2}{3}\sqrt{2} - \frac{1}{3} = 0.6095$

$u = \sqrt{1+x^2} = (1+x^2)^{1/2}$

$du = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x\,dx$

$= x(1+x^2)^{-1/2}\,dx$

$\sqrt{1+x^2}\,du = x\,dx$

$u\,du = x\,dx$

(b) $\int_1^3 \frac{1}{x^2} e^{2/x}\,dx = -\frac{1}{2}\int_2^{2/3} e^u\,du = -\frac{1}{2}\left[e^{2/3} - e^2\right]$

$u = \frac{2}{x} = 2x^{-1}$

$du = -2x^{-2}\,dx$

$-\frac{1}{2}\,du = \frac{1}{x^2}\,dx$

6. (a) $\int_1^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^3}\,dx = \lim_{b\to\infty} -\frac{1}{2}x^{-2}\Big|_1^b$

$= \lim_{b\to\infty}\left[-\frac{1}{2}\cancel{(b)^{-2}}^{0} - \left(-\frac{1}{2}\right)\right]$

$= \frac{1}{2}$

(b) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty} \int_1^b x^{-1/2}\,dx = \lim_{b\to\infty}\left[\frac{1}{\frac{1}{2}}x^{1/2}\Big|_1^b\right]$

$= \lim_{b\to\infty}\left[\cancel{2b^{1/2}}^{\infty} - 2\right] \to \infty$ divege

(c) $\int_{-\infty}^0 e^x\,dx = \lim_{a\to-\infty} \int_a^0 e^x\,dx = \lim_{a\to-\infty} e^x\Big|_a^0$

$= \lim_{a\to-\infty}\left[1 - \cancel{e^a}^{0}\right] = 1$

7. (a) $\int_0^1 \frac{\ln x}{x^3}\,dx$

$$= \lim_{h\to 0^+} \underbrace{\int_h^1 \frac{\ln x}{x^3}\,dx}_{I} = \lim_{h\to 0^+}\left[\frac{\ln h}{2h^2} - \frac{1}{4} + \frac{1}{4h^2}\right] \to \infty \text{ diverge}$$

$$I = -\frac{\ln x}{2x^2}\Big|_h^1 + \int_h^1 \frac{1}{2x^3}\,dx = -\frac{\ln x}{2x^2}\Big|_h^1 + \frac{1}{2}\left[\frac{1}{-2}x^{-2}\Big|_h^1\right]$$

$$\left(\begin{array}{ll} f(x)=\ln x & f'(x)=\frac{1}{x} \\ g'(x)=x^{-3} & g(x)=-\frac{1}{2}x^{-2} = -\frac{1}{2x^2} \end{array}\right)$$

$$= \frac{\ln h}{2h^2} + \frac{1}{2}\left(-\frac{1}{2}\right)\left[1 - h^{-2}\right]$$

$$= \frac{\ln h}{2h^2} - \frac{1}{4} + \frac{1}{4h^2}$$

(b) $\int_1^\infty \frac{\ln(x)}{x^3}\,dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^3}\,dx$

$$= \lim_{b\to\infty}\left[-\frac{\ln x}{2x^2}\Big|_1^b + \frac{1}{2}\left[\frac{1}{-2}x^{-2}\Big|_1^b\right]\right]$$

$$= \lim_{b\to\infty}\left[\cancelto{0}{-\frac{\ln b}{2b^2}} - \frac{1}{4}\left(\cancelto{0}{b^{-2}} - 1\right)\right] = \frac{1}{4}$$