

ECO 137 Quiz2 (Nov. 28. 2008)

1. Demand of apples is expressed as $D = 10I^{1.46}$ where I is income.
Derive income elasticity of demand. Show your calculation.

$$\ln D = \ln 10 + 1.46 \ln I$$

$$IED = \frac{d \ln D}{d \ln I} = 1.46$$

2. Find linear approximation to $f(t) = \ln(1 + t)$ about $t = 0$.

$$f(t) = \ln(1 + t), f(0) = 0$$

$$f'(t) = \frac{1}{1 + t}, f'(0) = 1$$

$$f(t) \approx 0 + 1 * (t - 0) = t$$

3. By using the result from 2, derive the approximate doubling time of 1 YTL when the annual interest rate is 10%. Show your calculation. ($\ln 2 = 0.7$)

$$\left(1 + \frac{p}{100}\right)^t = 2$$

$$t \ln \left(1 + \frac{p}{100}\right) = \ln 2 \Rightarrow t = \frac{\ln 2}{\ln \left(1 + \frac{p}{100}\right)}$$

Use the result from 2. $\ln \left(1 + \frac{p}{100}\right) \approx \frac{p}{100}$, $\ln 2 = 0.7$

$$t = \frac{0.7}{\frac{10}{100}} = 7$$

Therefore, the doubling time is 7 years.

4. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x^4}$. Find the following limits.

$$\lim_{x \rightarrow 0} f(x), \lim_{x \rightarrow 0} g(x), \lim_{x \rightarrow 0} [f(x) - g(x)], \lim_{x \rightarrow 0} \frac{f(x)}{g(x)},$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty, \quad \lim_{x \rightarrow 0} \frac{1}{x^4} = \infty$$

$$\lim_{x \rightarrow 0} [f(x) - g(x)] = \lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{x^4} \right] = \lim_{x \rightarrow 0} \left[\frac{x^2 - 1}{x^4} \right] = -\infty$$

$$\lim_{x \rightarrow 0} \left[\frac{f(x)}{g(x)} \right] = \lim_{x \rightarrow 0} \left[\frac{\frac{1}{x^2}}{\frac{1}{x^4}} \right] = \lim_{x \rightarrow 0} [x^2] = 0$$